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Does relative risk aversion vary with wealth? Evidence from households' portfolio choice data

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ABSTRACT

We test whether relative risk aversion varies with wealth using the Panel Study of Income Dynamics data in the U.S. Our analytical results indicate the following implications. For each household, there are two channels through which the risky share responds to wealth fluctuations, the income channel and the habit channel. Across households, there are heterogeneous responses through both the habit channel and the income channel. Finally, two potential misspecification problems on time-varying relative risk aversion arise when both heterogeneous responses through the habit channel and the responses through the income channel are ignored. Our main empirical findings are to show the importance of the income channel and the heterogeneous responses, and to provide strong evidence of relative risk aversion varying with wealth, after correcting two misspecification problems.

1. Introduction

The assumption that agent's relative risk aversion decreases with wealth is appealing because it provides an important mechanism that helps explain numerous economic phenomena. One popular way of generating time-varying relative risk aversion (hereafter TVRRA) with wealth is to assume habit formation preferences.[1] Macroeconomic models with habit formation preferences have been used to explain a variety of stylized macroeconomic facts that are hard to explain using theoretical models with standard constant relative risk aversion preferences. These facts include the research works by: Constantinides (1990), Jermann (1998), Campbell and Cochrane (1999), Boldrin et al. (2001), Otrok et al. (2002), and others for the equity premium; Boldrin et al. (2001) for the excess sensitivity of consumption to income; Shore and White (2002) for the equity home bias; Fuhrer (2000), Uribe (2002), and Christiano et al. (2005) for the hump-shaped response of aggregate variables to monetary shocks; and Ravn et al. (2006) for the countercyclical markups. However, despite the mounting literature that uses habit formation preferences, thus

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[1] The literature that directly tests time-varying risk aversion without assuming habit formation is limited. Guiso et al. (2013) find evidence that risk aversion increases substantially after the last financial crisis.

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automatically embodying a TVRRA assumption, there are only a few papers that test the empirical relevancy of TVRRA with U.S. micro-data and the evidence is mixed.

To answer this relevant question empirically, we first derive theoretical predictions of TVRRA on the relation between risky shares and financial wealth, then test these predictions using the U.S. data from the Panel Study of Income Dynamics (PSID). To derive the theoretical predictions between risky shares and financial wealth, we build a discrete-time portfolio choice model with time-varying external habit and time-varying labor income. Our emphasis on time-varying labor income is motivated by two empirical facts from the PSID data. The first fact is that the majority of households in the PSID data receive labor income and the second one is that a large portion of households in the PSID data experience a large drop of income. For example, about 41% of the households in the sample of households from 1984 to 1999 received income below 20% of their average income over time. Empirical facts on time-varying labor income are robust to using different data sets and are also documented in other countries [see, for example, Lillard and Willis, 1978; Lillard and Weiss, 1979; MaCurdy, 1982; Abowd and Card, 1989; Geweke and Keane, 2000; Guvenen, 2009 for U.S., and Le Blanc and Georgarakos, 2013 for five European countries].

Our model with time-varying labor income captures these two empirical observations in the data. More importantly, our model modifies the existing theoretical predictions about the relation between risky shares and financial wealth in a non-trivial way, which will be discussed below in detail.

The first contribution of our paper is to derive novel theoretical predictions of TVRRA on the relation between risky shares and financial wealth. The existing prediction in the literature argues that, when it becomes richer, the household will increase its risky share if the household has decreasing relative risk aversion in wealth [see Brunnermeier and Nagel, 2008]. Instead, our analytical solution changes such a conventional prediction in three dimensions.

First, our closed-form solution suggests that a household's risky share responds to wealth fluctuations through two channels. When the household does not experience a large drop of income, the risky share, as wealth accumulates, will increase because of positive habit. We call this the habit channel. At the same time, the risky share, in responding to financial wealth accumulations, will decrease because of positive income, which we call the income channel.[2] Thus, a misspecification problem may arise when the response through both the habit channel and the income channel is ascribed to the response through the habit channel alone. We call this an internal misspecification problem.

The second bias arises due to the heterogeneity in changes in households' income. Specifically, when they become richer, households with large income drops are likely to decrease their risky shares in response to wealth accumulations through the habit channel, while households without a large drop of income might increase their risky shares through the habit channel. Thus, in the presence of heterogeneous changes in income, households who have decreasing relative risk aversion in wealth may adjust their risky shares to wealth fluctuations in opposite directions. As a result, a misspecification problem, which we call an external misspecification problem, may arise when estimating over samples in which heterogeneous households are pooled together.

The changes in the above two dimensions hold when the external habit does not substantially deviate from its mean. The change we have made in the last dimension is about the impact of time-varying habit alone on the conventional prediction. Essentially, our theoretical solution shows that when external habit is far away from its mean, a household with habit formation preferences might not increase its risky share when it becomes richer, a prediction which is opposite to the conventional prediction. This change is important because it sheds light on the correct inference that one can get from her own empirical work.

Our second contribution is in the empirical analysis. In this regard, we define three different forms of TVRRA implications (hereafter TVRRAI): the strong form, the semi-strong form, and the weak form.[3] If the portfolio choice model with habit considers neither the aforementioned two channels (income channel and habit channel) for each household, nor the aforementioned heterogeneous responses through the habit channel across households, we label the key theoretical implication(s) from such a model as the strong form of TVRRAI. The strong form implies that households whose preferences could be represented by habit formation will, in the absence of income, increase their risky shares when their wealth increases, as discussed in Brunnermeier and Nagel (2008). If the portfolio choice model with habit considers the two channels but ignores the heterogeneity in income, we label the key theoretical implication(s) from such a model as the semi-strong form of TVRRAI. The semi-strong form controls for the internal misspecification problem, and implies that after controlling for the response through the income channel, the response through the habit channel should be positive. At last, if the portfolio choice model with habit considers both the two channels and the heterogeneity in income, we label the key theoretical implication(s) from such a model as the weak form of TVRRAI. That is to say, the weak-form controls both the internal and external misspecification problems. The weak-form implies that, after controlling for the response through the income channel and the impact of a large drop of income, the response through the habit channel in the group in which households experienced a large drop of income should be lower than that in the other group in which households did not experience a large drop of income.[4]

We empirically test the semi-strong form and the weak form of TVRRAI using the PSID data and compare the results with the strong form tested in Brunnermeier and Nagel (2008). We find evidence of the weak-form of TVRRAI in the 1984–1999

[2] We show these two channels analytically in Section 2.2.1.

[3] When a model imposes less restrictions, we say that the derived theoretical implication is stronger.

[4] Note that in the weak-form test, we do not use the sign of response of risk shares to wealth to make judgment about whether relative risk aversion is decreasing in wealth. The main justification behind our practice is our theoretical result that time-varying habit alone may break down the conventional prediction as we have discussed in the previous paragraph.

subsample and no evidence of the semi-strong form of TVRRAI in both the 1984–1999 and the 1999–2011 subsamples.[5] First, in the semi-strong form of TVRRAI, if the identification scheme builds on a model that does consider the two channels but ignores the heterogeneity so that the test corrects the internal but not the external misspecification problem, our estimates of semi-strong form testing are statistically insignificantly different from zero. This contrasts to Brunnermeier and Nagel (2008), who test the strong-form of TVRRAI and find significant negative responses. This comparison is in line with our theory, which states that controlling for the response through the income channel will increase the estimated response through the habit channel. Second, in the weak form of TVRRAI, if the identification scheme builds on a model that considers both the two channels and the heterogeneity, the tests will correct both the internal and the external misspecification problems. In this regard, our estimates of weak form testing are both economically and statistically significant; and they are in line with our benchmark theoretical predictions. Given that our estimates are significant and robust, we argue that they provide strong and clear evidence of TVRRA. In addition, the comparison between weak form testing results and strong form testing highlights the importance of isolating the heterogeneous impacts of large labor income drops in carrying out empirical tests of TVRRA.

Our paper is related to several strands of the existing literature. First, our paper contributes to the literature on the impact of wealth on households' portfolio choices.[6] Existing theoretical models with habit formation that abstract from labor income imply a positive relation between wealth and risky shares (see, for example, Constantinides, 1990; Campbell and Cochrane, 1999, and the references therein). In this paper, we show analytically how labor income affects such a relation. From the empirical study perspective, evidence is mixed in the literature. For example, Cappelletti (2012) uses Italian data and finds that, after controlling for the decision to enter and leave the risky asset market, wealth fluctuations do help us to explain changes in portfolio allocations. Brunnermeier and Nagel (2008) find a negative relation between risky shares and financial wealth in the U.S. data. Calvet et al. (2009) estimate the Brunnermeier–Nagel regression on Swedish data and find a positive relation after controlling for inertia. In this paper, we show that estimates which ignore the two aforementioned misspecification problems are likely to be biased down, as is predicted in our theory. Putting together, by carefully controlling for the impact of labor income on the relation between risky shares and financial wealth, we provide a mechanism that is able to reconcile the seeming conflict between the existing theoretical predictions of TVRRA from a model with habit formation preferences and the existing empirical findings.[7]

Second, our paper is related to the existing studies that test key theoretical predictions implied by habit using micro-data on consumption, which find mixed evidence of habit formation preferences. For example, Meghir and Weber (1996) and Dynan (2000) reject habit preference using US consumption data, and Chiappori and Paiella (2011) reject habit preference using Italian data. On the contrary, Ravina (2007) provides evidence of habit persistence in household consumption choices using panel data on U.S. credit-card account holders. Jimnez-Martn et al. (1998) find evidence of habit in tobacco consumption using Spanish Continuous Family Expenditure Survey. Carrasco et al. (2005) estimate the intra-temporal marginal rate of substitution using Spanish consumption panel data and find strong support of habit, while Browning and Collado (2007) do not find sufficiently strong degree of intertemporal dependence to support habit.

Finally, our paper is also related to the literature on how income affects households' portfolio choices. For example, Wachter and Yogo (2010) show in a life-cycle model that risky shares fall in normalized cash-on-hand (which corresponds to wealth in our paper) and rises in permanent income even if households have decreasing relative risk aversion in wealth. Our results instead indicate that the impact of *temporary* income also matters in terms of theoretical predictions of TVRRA on the relation between risky shares and financial wealth. First of all, our results show that controlling for the impact through the income channel may help explain the negative link found in Brunnermeier and Nagel (2008). Second, further controlling for the heterogeneous responses through the habit channel across households provides us evidence of TVRRA.

The rest of the paper is organized as follows. Section 2 presents the model and provides testable implications. Section 3 describes the data and reports the empirical results on both the weak form of TVRRAI and the semi-strong form of TVRRAI. Finally, Section 4 concludes the paper. The mathematical proofs are given in the Appendix.

2. Theoretical analysis

In this section, we present the benchmark model and the analytic solution to risky share, discuss the importance of time-varying income and time-varying habit, and derive testable implications of TVRRA. The theoretical model is a highly stylized portfolio choice model with a time-varying labor income and time-varying external habit. We consider the model for several reasons. First, the model delivers clear and testable predictions of TVRRA on how risky shares respond to wealth fluctuations.

[5] Thus, time-series data about asset holdings are either 2-year apart or 5-year apart. Hence, we divide the data into two subsamples: the 1984–1999 ($k=5$) subsample and the 1999–2011 ($k=2$) subsample.

[6] Lax (2000) uses habit formation to explain decreasing risky investment over the life cycle. Campbell and Viceira (2002, Section 6.1.3) shows analytically how subsistence level affects portfolio in a simple model with constant income. Other related paper includes Schroder and Skiadas (2002), Detemple and Karatzas (2003), Munk (2008), and Steffense (2011).

[7] It is worth mentioning that our mechanism is one of many possible ones that may be able to reconcile the seeming conflict. For example, one mechanism could be to control for the impact of inertia in portfolio adjustments as proposed in Calvet et al. (2009). They estimate the Brunnermeier–Nagel regression on their Swedish data set, with which they can control for inertia, and they find a positive relation when they estimate the regression by instrumental variables.

Second, our model captures the realistic feature that the majority of the US households do receive time-varying labor income. Third, the majority of macroeconomics models, if they assume habit formation preferences, contain these two elements.

2.1. The benchmark model

We incorporate time-varying income and time-varying habit in the basic framework of Samuelson (1969). In our model, a household carries financial wealth, W_t, from the last period, and receives labor income, Y_t, in the current period. The household can invest in two securities: a risky asset with return R_t and a risk-free asset with return R_f. We assume, as in Samuelson (1969), that the expected return and volatility of the risky return are constant. The household chooses consumption C_t and the share of post-consumption financial wealth $W_t+Y_t-C_t$ invested in the risky asset, α_t, to maximize

$$\max_{\{C_t,\alpha_t\}_{t=0}^{\infty}} \mathbb{E}_0[U(C_t,X_t,\delta,\gamma)] \quad \text{with } U(C,X,\delta,\gamma)=\sum_{t=0}^{\infty}\delta^t\frac{(C-X)^{1-\gamma}}{1-\gamma},$$

where $\mathbb{E}_0$ denotes the conditional expectation operator at time $t=0$, δ denotes the subjective discount factor, and X_t denotes the external habit. Note that, different from the model in Brunnermeier and Nagel (2008), our model assumes that the household receives time-varying labor income and time-varying habit.

In order to obtain the analytical solution, we make the following assumptions on labor income and habit, respectively

$$(Y_{t+1}-Y)=\kappa(Y_t-Y), \tag{2.1}$$

$$(X_{t+1}-X)=\eta(X_t-X), \tag{2.2}$$

where Y denotes the steady state value of labor income, X denotes the steady state value of external habit, and both $|\kappa|<1$ and $|\eta|<1$ are parameters. Alternatively, X can be interpreted as a constant subsistence level or a consumption commitment as in Chetty and Szeidl (2005). Our specification given in (2.1) and (2.2) assumes that both income and habit follow an AR(1) process. Our specification is a modified version of Campbell and Cochrane (1999) who specify a process for the log consumption surplus ratio. A recent work by Brandt and Wang (2003) assumes a similar stationary AR(1) process for log relative risk aversion.

Such a process enables us, when there is no uncertainty in the income and external habit processes, to derive a close-form solution of time-varying relative risk aversion implications on how risky shares respond to financial wealth fluctuations. The only uncertainly in this model is the risky return, on which the conditional expectation operator in the objection function is taken.

The household's financial wealth at the beginning of period $t+1$ is given by

$$W_{t+1}=(1+R_{p,t+1})(W_t+Y_t-C_t),$$

where $R_{p,t+1}=\alpha_t(R_t-R_f)+R_f$ denotes the return to the household's financial wealth portfolio.

It is not straightforward to derive the analytical solution to the risky share in our model. Our strategy to derive the analytical solution is as follows: transform the original model into one which has an analytical solution; and back up the solution to the risky share in the original model since both the original model and the transformed model should have the same amount of financial wealth invested in the risky asset. To successfully implement this strategy, we need to impose additional restrictions such as Eqs. (2.1) and (2.2). With these additional restrictions, we derive the analytical solution to the risky share in our benchmark model. The details are provided in Appendix A.1.

The solution to the risky share is given by:

$$\alpha_t^*=\left[1-\frac{X-Y}{\left(W_t-C_t+Y_t+\frac{(Y_t-Y)-(X_t-X)}{Z+R_f}\right)R_f}\right]\left[1+\frac{(Y_t-Y)-(X_t-X)}{(W_t-C_t+Y_t)(Z+R_f)}\right]. \tag{2.3}$$

where Z is defined as $Z=\left(\frac{1}{\kappa}-1\right)(1+R_f)$. Eq. (2.3) provides an analytical solution that enables us to discuss how the risky share responds to post-consumption financial wealth, how time-varying labor income affects the response, and how time-varying habit affect the inference from the sign of the response. It is worth mentioning that, if we set $Y_t=Y=0$ and $X_t=X$, the solution to the risky share, α_t, is simplified to

$$\alpha_t^*=\left[1-X/((W_t-C_t)R_f)\right], \tag{2.4}$$

which is the same as that in Brunnermeier and Nagel (2008). Therefore, our model nests the model in Brunnermeier and Nagel (2008) as a special case.

2.2. Specification and inference

In this section, we discuss two issues. First, we show that estimates of habit may be biased down if time varying labor income is ignored. In particular, those estimates are subject to the so-called misspecification problems, described below in Sections 2.2.1 and 2.2.2. To simplify the discussion of misspecification problems, we impose the restriction that external habit is constant in these two sections. Second, we show how time-varying habit may affect the inference from the sign of the estimates in Section 2.2.3. In discussing this inference issue, we assume that labor income is absent.

2.2.1. Internal misspecification

In this section, we show that a misspecification problem will arise if the income channel is ignored. To simplify the discussion, we set $X_t \equiv X$ and $Y_t \equiv Y$. In this case, Eq. (2.3) reduces to

$$\alpha_t^* = \underbrace{\left[1 - \frac{X}{(W_t - C_t + Y)R_f}\right]}_{\text{The habit channel}: +} + \underbrace{\left[\frac{Y}{(W_t - C_t + Y)R_f}\right]}_{\text{The income channel}: -}. \tag{2.5}$$

It is clear that α_t^* responds to the change of $W_t - C_t$ in two channels. We denote the term in the first bracket "the habit channel", and the one in the second bracket "the income channel". The sign "+" ("−") indicates that the risky share will increase (decrease) when the post-consumption financial wealth increases.[8] A positive habit X implies that the habit channel is positive and a positive income Y implies that the income channel is negative. As a result, ignoring the impact of labor income will bias down the estimates that are used to infer the time-varying relative risk aversion implications.

The internal misspecification problem arises when the response through two channels is ascribed as the response through the habit channel. To see this, it is easy to show from (2.5) that the following core regression equation holds true

$$\Delta\alpha_t^* = (\rho + \theta Y)\Delta w_t + \varepsilon_t, \tag{2.6}$$

where Δ denotes the first-order difference, $\rho = \frac{XW}{(W+Y)^2 R_f}$ and $\theta = \frac{-W}{(W+Y)^2 R_f}$, where $w_t \equiv \log(W_t - C_t)$, and ε_t follows identically independent normal distribution and is uncorrelated with Δw_t. Note that ρ is the parameter that catches the response of risky shares to financial wealth fluctuations through the habit channel. θ is the parameter that catches the response of risky shares to financial wealth fluctuations through the income channel. The detailed derivations of (2.6) are given in Appendix A.2.

If we remove labor income from the previous model, thus ignoring the impact of income on the relation, the core regression equation and the corresponding ordinary least squares (hereafter OLS) estimate are, respectively, given by

$$\Delta\alpha_t^* = \rho\Delta w_t + \varepsilon_t, \quad \text{and} \quad \tilde{\rho} = \left[(\Delta w_t)'(\Delta w_t)\right]^{-1}(\Delta w_t)'\left(\Delta\alpha_t^*\right).$$

However, if Eq. (2.6) is correctly specified, it follows straightforwardly based on the theory for OLS estimates that

$$\mathbb{E}(\tilde{\rho}) = \left[(\Delta w_t)'(\Delta w_t)\right]^{-1}(\Delta w_t)'(\rho - \theta Y)\Delta w_t = \rho - \theta Y \le \rho, \tag{2.7}$$

which means that $\tilde{\rho}$ is under-estimated. Thus, a misspecification problem arises when the estimate of $(\rho - \theta Y)$ is ascribed as the estimate of ρ. Moreover, if labor income has a strong impact, i.e., θY is large, $\tilde{\rho}$ may be close to zero or negative even though the true value of ρ is still positive. Hence, the fact that $\tilde{\rho}$ is close to zero or negative does not necessarily imply that micro-data do not support the TVRRA assumption, because it does not necessarily mean that ρ is negative or zero.

2.2.2. External misspecification: heterogeneity

In this section, we set $X_t \equiv X$ but allow time-varying income as in Eq. (2.1). Households with different changes in income may adjust differently their risky shares to financial wealth fluctuations through the habit channel. When households do not have a large drop of income, they increase their risky shares as the optimal response to financial wealth accumulations through the habit channel. However, when households have a large drop of income, they may decrease their risky shares as the optimal response to financial wealth accumulations through the habit channel. To see the heterogeneous responses, note that when Y_t is time-varying, the response of α_t^* through the habit channel is given by

$$\underbrace{\left[1 - \frac{X}{\left(W_t - C_t + Y_t + \frac{Y_t - Y}{Z + R_f}\right)R_f}\right]\left[1 + \frac{Y_t - Y}{(W_t - C_t + Y_t)(Z + R_f)}\right]}_{\text{The habit channel}}, \tag{2.8}$$

which shows that if Y_t is far below Y, it is likely that the sum in the second parenthesis becomes negative.[9] This implies that the response through the habit channel can be negative. In this case, the conventional wisdom that risky shares are increasing in financial wealth through the habit channel may break down in the presence of a large drop of income.

As a result, there could be two different groups of households. Households in the first group have a large drop of income and respond negatively, in terms of adjusting their risky shares, to financial wealth accumulations through the habit channel. Households in the second group do not have a large drop of income and they respond positively. If we run regressions with a sample that pools the two heterogeneous groups together, the associated estimate is misspecified and it is likely to be insignificant. We label this misspecification problem as the external misspecification problem.

Similarly, there are heterogeneous responses to financial wealth fluctuation on risky share through the income channel. To see the heterogeneous responses, note that when Y is time-varying, the response of α_t^* through the income channel is

[8] One thing worth mentioning is, even though α_t^* is decreasing in post-consumption financial wealth through the second channel, $\partial\alpha_t^*/\partial Y$ is still positive: the higher labor income the household has, the larger the α^* will be.

[9] Note that we have imposed $X_t \equiv X$.

given by

$$\underbrace{\left[\frac{Y}{\left(W_t-C_t+Y_t+\frac{Y_t-Y}{Z+R_f}\right)R_f}\right]\left[1+\frac{Y_t-Y}{(W_t-C_t+Y_t)(Z+R_f)}\right]}_{\text{The income channel}}. \tag{2.9}$$

Households with different changes in income may adjust differently their risky shares to financial wealth fluctuations through the income channel. When households do not have a large drop of income, they decrease their risky shares as the optimal response to financial wealth accumulations through the income channel. However, households who have a large drop of income adjust positively their risky shares to financial wealth accumulations through the income channel.

2.2.3. *Time-varying habit and inference*

In this section, we discuss how time-varying habit will affect the conventional prediction about the relationship between relative risk aversion and the sign of the response of risky shares to financial wealth. To focus on the point, we impose that $Y_t \equiv Y \equiv 0$. In this case, Eq. (2.3) reduces to

$$\alpha_t^* = \left[1-\frac{X}{\left(W_t-C_t-\frac{X_t-X}{Z+R_f}\right)R_f}\right]\left[1-\frac{X_t-X}{(W_t-C_t)(Z+R_f)}\right]. \tag{2.10}$$

Clearly, the necessary and sufficient condition for the last term on the right hand side of Eq. (2.10), $\left[1-\frac{X_t-X}{(W_t-C_t)(Z+R_f)}\right]$, to be positive, is $(X_t-X) < (W_t-C_t)(Z+R_f)$. That means that when habit does not dramatically deviate above from the mean, we have the conventional wisdom that households with habit formation preferences will increase their risky shares when they accumulate financial wealth. This is the same theoretical prediction as in Brunnermeier and Nagel (2008) which assume time-invariant habit. In other words, allowing time-varying habit does not fundamentally change the positive prediction in the literature if habit moves closely around the mean.

However, if X_t is far above X, it becomes possible that the response of risky shares to financial wealth accumulations through the habit channel becomes negative. This happens even if households have not suffered from a large drop of income. In real life, it seems intuitive that habit will have a larger change over a longer period of time. In this case, it seems reasonable to argue that the chance that the response becomes negative will increase over longer time horizons. This possibility is important. It implies that we cannot, unless we have controlled the impact of time-varying habit, infer whether relative risk aversion is decreasing in financial wealth or not from the sign of the response of risky shares to financial wealth.

2.3. *Testable predictions*

We have shown that incorporating time-varying labor income matters in terms of identification and incorporating time-varying habit matters based on inference. We now derive empirical tests of the theoretical predictions of TVRRA by controlling the response through the income channel for each household and/or the heterogeneous responses through the habit channel across households, i.e., testing the semi-strong form and the weak form of TVRRAI.

2.3.1. *Semi-strong form of TVRRAI*

Given the internal misspecification problem, we run regression to obtain the estimates of the response through the habit channel, ρ. We consider the following testable hypothesis to test the semi-strong form of TVRRAI

$$\mathbb{H}_0^{\text{Semi-Strong}}: \rho > 0. \tag{2.11}$$

Mathematically, without considering the impact of a large drop of income (or as in the case of $Y_t \equiv Y$), our analysis in Section 2.2.1 indicates that $X > 0$ implies that $\rho > 0$. That is to say, a positive estimate of ρ suggests that habit formation preferences are in line with portfolio choice data. This is the same as in Brunnermeier and Nagel (2008). Intuitively, an increase in financial wealth, for example, should lead to a temporary decrease in relative risk aversion and an increase of the risky share if households have habit formation preferences.

2.3.2. *Weak form of TVRRAI*

Given the aforementioned internal and external misspecification problems, we design the following test to examine the weak form of TVRRAI. We divide households in each subsample into two groups: households in the first group, $i=1$, experienced a large drop of income and households in the second group, $i=2$, did not experience a large drop of income. Second, for each group, we obtain an estimate of ρ_i, $i=1,2$. Third, our testable hypothesis for habit formation preference is,

$$\mathbb{H}_0^{Weak}: \rho_2-\rho_1 > 0. \tag{2.12}$$

Instead of imposing $\rho_2 > \rho_1 > 0$, we test the difference of ρ's across groups in each subsample. The main reason for not testing $\rho_i > 0$ is as follows. As we have discussed in Section 2.2.3, when X_t dramatically deviates from its mean, the response of risky shares to financial wealth fluctuations through the habit channel would be negative even if households have not experienced a large drop of income. When this happens, we cannot, unless we have controlled the impact of time-varying habit, infer whether relative risk aversion is decreasing in financial wealth or not from the sign of either ρ_1 or ρ_2. In our empirical analysis in Section 3, the test of the weak form of TVRRAI uses the $k=5$ subsample.[10] During the five-year interval between any two observations for each household, it is possible that habit has changed a lot. Because of possible large changes of habit over 5 years, we test the inequality given in (2.12).[11]

3. Empirical analysis

In this section, we present the data, the benchmark regression models, and the empirical results. When we present the empirical results, we first explore the model that considers both channels but with constant income (semi-strong form of TVRRAI); and then investigate our benchmark model with time-varying labor income that considers both the habit channel and the income channel, and the heterogeneity (the weak form of TVRRAI). In addition, it is well known that PSID data, micro-data from surveys, about wealth and risky shares contain measurement errors and OLS estimates are inconsistent. Thus, we skip the discussion of inconsistent OLS estimation results and only focus on the two-stage least square (TSLS) estimates.

3.1. Data

In this section, we give a brief introduction about the PSID data, the key variables used in the empirical analysis, and our sampling criteria. First, PSID is a national study of socioeconomics over lifetimes and across generations. The study began in 1968 with a nationally representative sample of over 18,000 individuals living in 5000 families in the United States. The PSID data cover employment, income, wealth, expenditures, health, marriage, childbearing, education, and many others. In terms of the data used in this paper, the PSID data set contains many household characteristics and income data annually before 1998 (ending in 1997) and biannual afterwards (starting in 1999). The households' asset holdings are measured in years 1984, 1989, 1994, 1999, 2001, 2003, 2005, 2007, 2009, and 2011. Thus, time-series data about asset holdings are either 2-year apart or 5-year apart. Hence, we divide the data into two subsamples: the 1984–1999 ($k=5$) subsample and the 1999–2011 ($k=2$) subsample.

Second, the definitions of key variables are quite standard. In particular, liquid assets are given by the sum of risk-free assets and the holdings of stocks and mutual funds. Subtracting other liabilities from liquid assets yields liquid wealth. Financial wealth is defined as the sum of liquid wealth, equity in a private business, and home equity. As is common in the literature, risk-free assets are defined as the sum of cash-like assets and holdings of bonds; and risky assets include the holdings of stocks and mutual funds, equity in a private business, and home equity. The risky share is the holdings of risky wealth divided by financial wealth.

The income in this paper refers to the labor income of the household. The values for "labor income" represent the summation of head's labor income and wife's labor income. The values for head's labor income represents the summation of the following variables: labor part of farm income, labor part of business income, head's wages income, head's bonuses, overtime, commissions, head's income from professional practice or trade, labor part of market gardening income, and labor part of roomers and boarders income. If the wife had any income from farming, business, market gardening, or roomers and boarders, labor-asset splits were made following the same rules as those for the head. The labor portion of such income is included in wife's labor income. We focus on the labor income because wealth in our theoretical model is interpreted as financial wealth.

Very last, we apply the same sampling criteria as in Brunnermeier and Nagel (2008). In this regard, we first require a certain minimum level of wealth. In practice we follow Brunnermeier and Nagel (2008) by setting a wealth lower bound of $10,000. Note that our major results are not sensitive in the sense that they still hold when we lower the bound to $5000. Second, we require that the martial status of the family unit head remained unchanged from $t-k$ to t; that no assets were moved in or out as a consequence of a family member moving into or out of family unit; and that the retirement status has not changed from $t-k$ to t. As in Brunnermeier and Nagel (2008), we deflate all income and wealth data by the consumer price index (CPI) into December 2001 dollars. Table 1 provides some summary statistics of labor income, liquid wealth, and financial wealth.

3.2. Benchmark regression of semi-strong form testing

In testing the semi-strong form of TVRRAI, i.e., testing the inequality in (2.11), we estimate, in both the 1984–1999 subsample and the 1999–2011 subsample, the following equation

$$\Delta_k \alpha_{t,j} = \beta q_{t-k,j} + \gamma \Delta_k h_{t,j} + \rho \Delta_k w_{t,j} + \vartheta y_{t,j} \Delta_k w_{t,j} + \varepsilon_{t,j}. \tag{3.1}$$

Here Δ_k denotes the change over the k-year period. For example, $\Delta_k \alpha_{t,j}$ means the change of $\alpha_{t,j}$ and $\Delta_k w_{t,j}$ means the change of $w_{t,j}$ (wealth) in the past k years. $q_{t-k,j}$ is a vector of household characteristics and the fixed time effects for the jth household. For example, it includes a broad range of variables related to the life cycle, background, and financial situation of

[10] The notation of $k=5$ means that any two observations for the same household in the sample are 5 years apart. We explain in detail in Section 3.1.

[11] Nevertheless, our testable hypothesis is still reasonable in this case as long as habit across households changes roughly in the same fashion over time.

the household. The vector $\Delta_k h_{t,j}$ contains variables that capture major changes in household characteristic or asset ownership. For example, it includes changes in family size, changes in the number of children, and sets of dummies for house ownership, business ownership, and nonzero labor income at t and $t-k$. The inclusion of these additional variables serves

Table 1
Summary statistics.

Variables	Mean	Tenth Percentile	Median	Ninetieth Percentile	N
All households, 1984–1999 ($k=5$)					
Labor Income	75,691	8145	62,446	137,205	3324
Financial wealth	458,003	38,107	207,882	893,485	3324
Liquid wealth	168,281	1199	55,475	369,449	3324
All households, 1999–2011 ($k=2$)					
Labor Income	87,861	13,706	67,307	161,933	9406
Financial wealth	548,006	35,314	236,500	1,095,356	9406
Liquid wealth	199,910	257	62,548	460,526	9406

Notes:
1. The values for "labor income" represent the summation of head's labor income and wife's labor income.
(a) The values for head's labor income represent the summation of the following variables: labor part of farm income, labor part of business income, head's wages income, head's bonuses, overtime, commissions, head's income from professional practice or trade, labor part of market gardening income, and labor part of roomers and boarders income.
(b) If the wife had any income from farming, business, market gardening, or roomers and boarders, labor-asset splits were made following the same rules as those for the head. The labor portion of such income is included in wife's labor income.

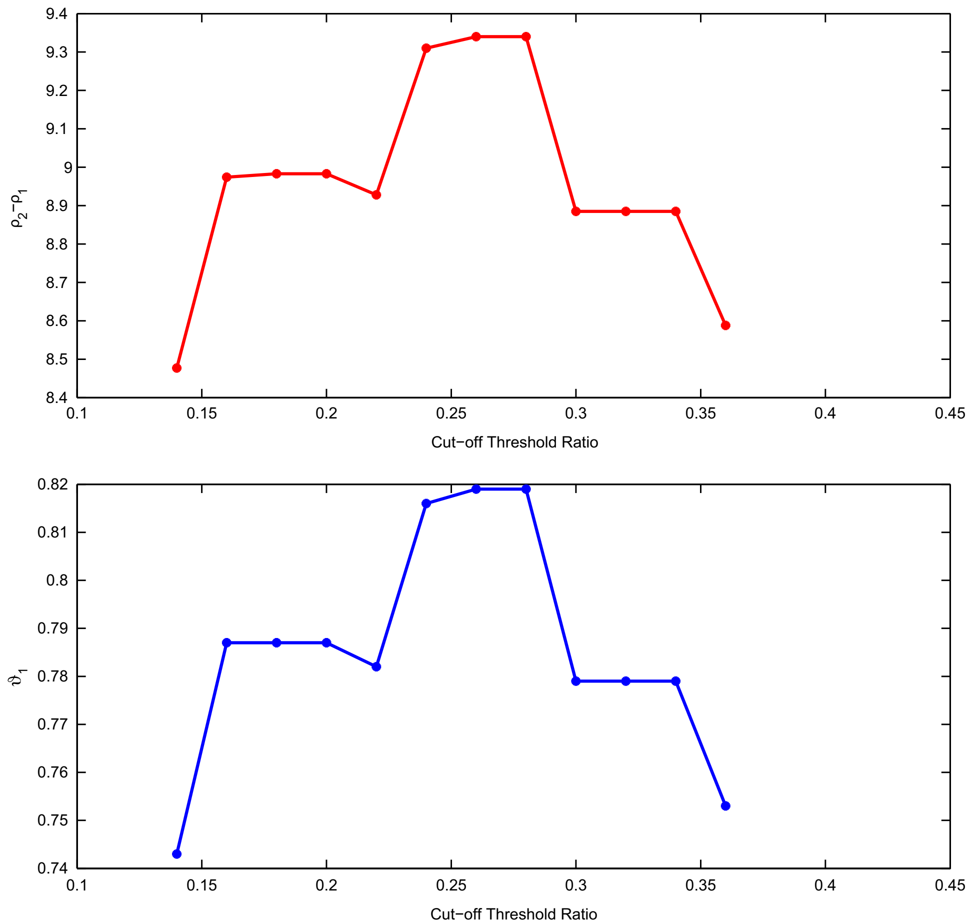

Fig. 1. Results for sensitivity analysis. *Note*: (1) In both panels, the horizontal axis represents the value we set for the threshold ratio that is used to divide the sample into two groups. (2) In the upper panel, the vertical axis represents the difference between $\hat{\rho}_2$ and $\hat{\rho}_1$, where ρ measures the response of financial risky shares to financial wealth fluctuations through the income channel and the subscript i means the ith group. In drawing the figure in this panel, we set $\hat{\rho}_i$ at zero if it is not statistically significantly different from zero at the 5% significant confidence interval. (3) In the bottom panel, the vertical axis represents $\hat{\vartheta}_1$, where ϑ measures the response of financial risky shares to financial wealth fluctuations through the income channel and the subscript i means the i-th group. All $\hat{\vartheta}_2$ are not statistically significantly different from zero.

the purpose of controlling for some important econometric issues, such as life-cycle effects and preference shifters, and idiosyncratic versus aggregate wealth changes. We use labor income in our empirical analysis of financial risky shares, so that Y_t denotes the labor income of the household and we set $y_t = \log(Y_t)$.

3.3. Benchmark regression of weak form testing

In testing the weak form of TVRRAI, i.e., testing the inequality in (2.12), we take two steps. In the first step, we divide the households in the subsample into two groups. In the first group, $i=1$, households' current income is below a threshold ratio of their time-series averages. The remaining households enter the second group, $i=2$. In the second step, we estimate, for each group in the subsample, the following equation

$$\Delta_k \alpha^i_{t,j} = \beta^i q^i_{t-k,j} + \gamma^i \Delta_k h^i_{t,j} + \rho_i \Delta_k w^i_{t,j} + \vartheta^i y^i_{t,j} \Delta_k \left(w^i_{t,j} \right) + \varepsilon^i_{t,j}, \quad i = 1, 2. \tag{3.2}$$

where the superscript i denotes the ith group. In the benchmark exercise, we set the threshold ratio at 20%. To see the impact of the threshold ratio on the results, we conduct a sensitivity analysis by changing the values for threshold ratio from 14% to 36% and the analysis results are displayed in Fig. 1.

Finally, compared to Eq. (10) in Brunnermeier and Nagel (2008), we introduce the term $y_{t,j} \Delta_k (w_{t,j})$ in Eq. (3.1) [or the term $y^i_{t,j} \Delta_k \left(w^i_{t,j} \right)$ in Eq. (3.2)] in order to get the estimate of ρ (or ρ_i). Since this additional term is the only difference between Eq. (3.1) [or Eq. (3.2)] and Eq. (10) in Brunnermeier and Nagel (2008), all econometric issues except the instruments that have been addressed in Brunnermeier and Nagel (2008) are handled in the same way.

3.4. Instruments

A key issue about TSLS estimates is the choice of instruments. The identification requirement is that the instruments, *IV*s, are (partially) correlated with $\Delta_k w_t$, but not correlated with the error terms. Brunnermeier and Nagel (2008) choose quantile dummies for income growth from $t-k$ to t [similar to Dynan (2000), in a different application], and inheritance receipts [as in Meer et al. (2003)] between $t-k$ and t as instruments. One reason is, as they argue, that these instruments are based upon survey questions that are different from those for the components of w_t. Hence, it is reasonable to assume that the elements of their *IV*s are uncorrelated with measurement errors.

However, we report our testing results with a different set of instruments in Tables 3 and 4. The reason is that Brunnermeier and Nagel (2008) instruments are in general weak instruments when we run our regressions in testing the weak form. For example, the Cragg–Donald Wald F statistics using the bottom 20% group in the 1984–1999 subsample is 5.01, and we cannot reject the hypothesis of weak instruments at the 30% significant level.

To deal with the weak instrument issue, we choose different instruments. In particular, our instrument, *IV*, is the difference between the growth rate of the household's labor income and the growth rate of the household's liquid assets. The instrumental variable, *IV*, is given by $IV = \log(labfw/llabfw)$, where $labfw = hdlabinc5/(fw + svodbt)$ and $llabfw = lhdlabinc5/\ (lfw + lsvodbt)$. Here $hdlabinc5$ and $lhdlabinc5$ denote head's labor income in the current period and in the past period, respectively, fw and lfw denote the liquid wealth in the current period and in the past period, respectively, and $svodbt$ and $lsvodbt$ denote the dollar value of other debts in the current period and in the past period, respectively (other debts comprise nonmortgage debt such as credit card debt and consumer loans). Thus, $(fw + svodbt)$ and $(lfw + lsvodbt)$ denote liquid assets in the current period and in the past period, respectively. Note that, according to Cai et al. (2006), one IV is sufficient in regressions like ours. Finally, to be consistent, we use the same set of instrument in discussing empirical results of all three forms.

It seems clear that our instrument, by its nature, is related to the financial wealth fluctuations. The partial R^2 of the instrument is close to 1, which suggests that the instrument explains a large fraction of variation in wealth changes. This instrument is arguably strong as indicated by the results in Tables 3 and 4. In terms of weak form testing, the results in panel (a) of Table 4 show that the instrument has a significant partial correlation with changes in log financial wealth. The instrument is highly significant, with p-values smaller than 3%. In addition, the value of the F statistics is above Stock–Yogo weak identification critical values, rejecting the hypothesis that our *IV* is weak. In particular, the Cragg–Donald Wald F statistics associated with the first group is 7.2 is larger than the 20% Stock–Yogo weak *ID* test critical value.

3.5. Semi-strong form: importance of controlling for the income channel

With the new and the same instrument as that used in the weak form testing, we report the main results in Table 3. In general, we find no responses of financial risky shares to financial wealth fluctuations in our semi-strong form testing results in both the 1984–1999 and the 1999–2011 subsamples. For example, the estimates of the responses are -1.465 and -2.654 in the 1984–1999 and the 1999–2011 subsamples, respectively; and both of them are statistically insignificantly different from zero.

In contrast, Brunnermeier and Nagel (2008) find generally negative response of the financial risky share to the wealth fluctuations.[12] To replicate their strong form testing results, we estimate the following equation with both subsamples and

[12] This is true even after we extend their 1999–2003 subsample to the 1999–2011 subsample.

report the results in Table 3:

$$\Delta_k \alpha_{t,j} = \beta q_{t-k,j} + \gamma \Delta_k h_{t,j} + \rho \Delta_k w_{t,j} + \varepsilon_{t,j},$$

Even though our TSLS estimates are quantitatively different from their TSLS estimates mainly because we use different instrumental variables from theirs, both our TSLS estimates and theirs are negative and statistically significantly different from zero.

In addition, we find that the response of risky shares to financial fluctuations through the income channel is positive. In particular, we have $\hat{\vartheta} = 0.116$ in the 1984–1999 subsample and $\hat{\vartheta} = 0.219$ in the 1999–2011 subsample. Although their signs are opposite to what the corresponding theory has predicted, the estimates are not statistically significantly different from zero.

We have obtained three key results: significant estimates of ρ in strong form testing; and insignificant estimates of ρ and insignificant but positive estimates of ϑ in semi-strong form testing. These results are robust to the choice of Brunnermeier and Nagel (2008) instruments. For example, when we instead use Brunnermeier and Nagel (2008) instruments, our empirical results show that in both subsamples (1) these instruments are not weak; (2) in the semi-strong form testing, both $\hat{\rho}$ and $\hat{\vartheta}$ are not statistically significantly different from zero; and (3) in the strong form testing, $\hat{\rho}$s are negative and statistically significantly different from zero.

Put together, the comparison between the semi-strong and strong forms of TVRRAI shows that controlling for the response through the income channel raises the estimate of ρ, consistent with the implication of our theoretical model with constant labor income that omitting the impact of labor income channel biases downward the estimate of ρ.

3.6. *Weak form: evidence of TVRRA*

The TSLS regression results about the weak form testing are presented in Table 4 and Fig. 1. We discuss the group sizes, the response through the habit channel, the response through the income channel, and the robustness check, one by one.

3.6.1. *Group sizes*

To obtain the two groups, we need to compare households' current labor income to their average. We calculate average labor income as:

$$\overline{y}_{t,j}|_1 = \left(y_{t,j} + y_{t-1,j} + y_{t-2,j}\right)/3, \tag{3.3}$$

For the 1999–2011 subsample, we cannot calculate $\overline{y}_{t,j}|_1$ because we only have biannual data for this subsample and data for $y_{t-1,j}$ is missing. As a result, we only report the empirical results about testing the weak form of TVRRAI in the 1984–1999 subsample.

It immediately follows that a big portion of households in our subsample suffered a large drop of income. 509 out of 1238 households in the 1984–1999 subsample, 41 percent, had the current labor income below 20% of their individual time average. It is worth mentioning that, as far as we know, the fact that a large portion of households in the 1984–1999 subsample (in the U.S.) had received labor income lower than the 20% of a specific average of their individual labor income is first documented in this paper.

Those empirical facts on time-varying labor income are well-documented and are robust to using different data sets. For example, MaCurdy (1982) uses PSID and finds that the change of earnings can be approximated by 2nd order moving average. Abowd and Card (1989) analyze changes in individual earnings over time using three different data set (PSID, National Longitudinal Survey of Men 49–59, and a sample of Seattle and Denver Income Maintenance Experiment). Among papers that studies time variations in labor income in other countries, Le Blanc and Georgarakos (2013) estimate income process using panel data for five European countries (Germany, Spain, Italy, France, and U.K.). Papers in a special issue of the Review of Economic Dynamics in 2010 on "Cross Sectional Facts for Macroeconomists" show several dimensions for economic inequality, including income, over time and over the life cycle. Earnings process for nine countries (US, Canada, UK, Germany, Italy, Spain, Sweden, Russia, and Mexico) have been estimated.

The variation in income has been largely attributed to measurable variables, permanent shocks and transitory shocks, and cyclical variation. Meghir and Pistaferri (2004), while confirming that permanent and transitory shocks are important components of changes in income, find unobserved heterogeneity in the variances. Moffitt and Gottschalk (2002) show that variance of permanent component has being raising since 1970s while the variance of transitory component raised until 1990s and started to decline afterwards.

It is worth mentioning that there are no substantial differences between the two subgroups over some key variables such as wealth, income, age, education, marital status, children, as shown in Table 2.

3.6.2. *Responses through the habit channel*

Our theoretical prediction that $\rho_2 > \rho_1$ is supported in the data. First, we have only one endogenous regressor and one instrument in our regression specifications. Thus, our specifications are exactly identified and the *p*-values associated with the over-identification tests are always zero. Second, in the first group in which households suffered from a large drop in labor income, the response of financial risky shares to financial wealth fluctuations is -8.983 percentage points. The estimate is not only statistically significant, but also economically significant. While in the second group in which households did not suffer from a large drop of income, the response is positive, but not statistically significantly different from zero. Third, it is thus reasonable to argue that the response of financial risky shares to wealth fluctuations in the first group is 8.983 percentage points smaller than the corresponding response in the second group. This difference is both statistically and economically

Table 2
Summary statistics of two subgroups in the 1984–1999 subsample.

Variables	Below 20% ($N=628$)		Above 20% ($N=817$)	
	Mean	STD	Mean	STD
Wealth	810,035	2,325,078	599,676	1,146,705
Income	89,563	80,797	101,917	134,162
Age	51.1	11.4	48.9	11.1
Education1	0.92	0.27	0.94	0.25
Education2	0.45	0.50	0.60	0.49
Martial status	1.54	1.04	1.29	0.76
Children	0.72	1.03	0.74	1.06

Notes:
1. Education1: dummy for high school completion (1=yes)
2. Education2: dummy for bachelors degree (1=yes)
3. Marital status: 1=married, 2=never married, 3=widowed, 4=divorced, 5=separated.

Table 3
Semi-strong and strong form testing: TSLS results.

	$k=5$ (1984–1999)		$k=2$ (1999–2011)	
	Semi-strong	Strong	Semi-strong	Strong
Dependent variable:				
Δ_k log financial wealth$_t$				
Key explanatory variables:				
Instrument variable	-0.006^{**}	-0.353^{***}	-0.004^{**}	-0.321^{***}
(b) Second stage results	$k=5$ (1984–1999)		$k=2$ (1999–2011)	
	Semi-strong	Strong	Semi-strong	Strong
Dependent variable:				
Δ_k Proportion of financial wealth invested in stocks, mutual funds, equity in a private business, and home equity				
Explanatory variables[a]:				
Δ_k log financial wealth$_t$	-1.465	-0.167^{***}	-2.654	-0.178^{***}
$y^i_{t,j}\Delta_k\left(w^i_{t,j}\right)$	0.116	–	0.219	–
Weak instrument test	11.9	453	18.2	1283
Significance (%)	15	5	10	5
N	1238	1264	3664	3725

Notes:
1. For the semi-strong form test, the regression equation is given by

$$\Delta_k\alpha_{t,j}=\beta q_{t-k,j}+\gamma\Delta_k h_{t,j}+\rho\Delta_k w_{t,j}-\vartheta y_{t,j}\Delta_k w_{t,j}+\varepsilon_{t,j}.$$

2. For the strong form test, the regression equation is given by

$$\Delta_k\alpha_{t,j}=\beta q_{t-k,j}+\gamma\Delta_k h_{t,j}+\rho\Delta_k w_{t,j}+\varepsilon_{t,j}.$$

3. The instrumental variable, IV, is given by $IV=\log(labfw/llabfw)$, where $labfw=hdlabinc5/(fw+svodbt)$ and $llabfw=lhdlabinc5/(lfw+lsvodbt)$. Here $hdlabinc5$ and $lhdlabinc5$ denote head's labor income in the current period and in the past period, respectively, fw and lfw denote the liquid wealth in the current period and in the past period, respectively, and $svodbt$ and $lsvodbt$ denote the dollar value of other debts in the current period and in the past period (other debts comprise nonmortgage debt such as credit card debt and consumer loans), respectively. Thus, $(fw+svodbt)$ and $(lfw+lsvodbt)$ denote liquid assets in the current period and in the past period, respectively.
4. Other explanatory variables include preference shifters, life-cycle controls, and year-region dummies, for details, see Section 3.1.
5. Heteroskedasticity and autocorrelation-robust standard errors are used to judge the significance of estimates.
** Means that the estimate is statistically significantly different from 0 at the 5% significance level.
*** Means that the estimate is statistically significantly different from 0 at the 1% significance level.

significant. Put all together, we argue that the TSLS results do provide evidence of TVRRA since they are in line with the theoretical predictions about relative risk aversion of a portfolio choice model with habit, given that our instrument is strong and the difference is both statistically significant and economically significant.

3.6.3. *Responses through the income channel*

The responses financial risky shares to wealth fluctuations through the income channel are also in line with our benchmark theoretical implications. From Eq. (2.9) and following the same logic as we have used to derive the prediction, inequality (2.12), it is straightforward to obtain an important implication:

$$\mathbb{H}_0^{Weak,Income}: \vartheta_2 - \vartheta_1 > 0. \tag{3.4}$$

We do not formally write this as a testable hypothesis in Section 2.3.2 because we focus on the estimates of the responses through the habit channel. In the first group, the estimate of the response of financial risky shares to financial wealth fluctuations through the income channel is -0.787, an estimate that is statistically significant and arguably also economically significant. In the second group, the estimate is -0.496, an estimate that is not statistically significant. Put these two observation together, it is reasonable to argue that the estimates are in line with the inequality (3.4).

Our results about the estimates of responses through the income channel provide strong justification to the use of our benchmark theoretical model to analyze the TVRRA issue. In Section 3.5, we have shown that it is important to isolate the response of financial risky shares to financial wealth fluctuations through the income channel. By controlling such a response, we obtain quite different estimates from what we have obtained if we do not control for the income channel. However, two issues remain there: the estimates of the response through the income channel are not statistically significantly different from zero and the sign seems to be opposite to what the corresponding theory has predicted. By considering the heterogeneous responses, we now have the right signs for the estimates in both groups and the statistically significant estimate in the first group. This is in line with an even stronger implication, such as $\vartheta_2 - \vartheta_1 > 0$ & $\vartheta_1 < 0$ & $\vartheta_2 > 0$. For example, we have $\hat{\vartheta}_1 < 0$; and even though $\hat{\vartheta}_2$ is not statistically significantly different from zero, it has the right sign.

Put together, our estimates of the responses through the income channel provide another important piece of evidence about (1) the importance of considering the responses through the income channel and (2) the importance of considering the heterogeneous responses throughout households.

3.6.4. Robustness check

We have done robustness checks in several dimensions. First, we change the 20% threshold value from 14% to 36% and we obtain similar evidence of TVRRA. In Fig. 1, the horizontal axis represents the threshold ratio that is used to divide the 1984–1999 subsample into two groups. In our empirical exercise, all $\hat{\rho}_2$'s are not significantly different from zero at the 5%

Table 4
Weak form testing: TSLS results.

(a) First stage results	k=5 (1984–1999)	
	Below 20%	Above 20%
Dependent variable:		
Δ_k log financial wealth$_t$		
Key explanatory variables:		
Instrumental Variable: *IV*	−0.007**	−0.006*
(b) Second stage results	k=5 (1984–1999)	
	Below 20%	Above 20%
Dependent variable:		
Δ_k Proportion of financial wealth invested in stocks, mutual funds, equity in a private business, and home equity		
Key explanatory variables[a]:		
Δ_k log financial wealth$_t$	−8.983**	5.359
	(3.792)	(7.631)
$y^i_{t,j}\Delta_k\left(w^i_{t,j}\right)$	0.787**	−0.496
	(0.335)	(0.686)
Weak instrument test	7.2	7.7
Significance (%)	20	20
N	509	729

Notes:
1. The benchmark regression equation is given by

$$\Delta_k \alpha^i_{t,j} = \beta^i q^i_{t-k,j} + \gamma^i \Delta_k h^i_{t,j} + \rho_i \Delta_k w^i_{t,j} - \vartheta^i y^i_{t,j} \Delta_k \left(w^i_{t,j}\right) + \varepsilon^i_{t,j}, i = 1, 2.$$

2. The instrumental variable, *IV*, is given by $IV = \log(labfw/llabfw)$, where $labfw = hdlabinc5/(fw + svodbt)$ and $llabfw = lhdlabinc5/(lfw + lsvodbt)$. Here *hdlabinc*5 and *lhdlabinc*5 denote head's labor income in the current period and in the past period, respectively, *fw* and *lfw* denote the liquid wealth in the current period and in the past period, respectively, and *svodbt* and *lsvodbt* denote the dollar value of other debts in the current period and in the past period (other debts comprise nonmortgage debt such as credit card debt and consumer loans). Thus, $(fw + svodbt)$ and $(lfw + lsvodbt)$ denote liquid assets in the current period and in the past period, respectively.
3. Other explanatory variables include preference shifters, life-cycle controls, and year-region dummies, for details, see Section 3.1.
4. Heteroskedasticity- and autocorrelation-robust standard errors are used to judge the significance of estimates.
** Means that the estimate is statistically significantly different from 0 at the 5% significance level.
* Means that the estimate is statistically significantly different from 0 at the 10% significance level.

significant confidence interval. Thus, we set them to be zero and the vertical axis in the upper panel in Fig. 1 represents the resulted $\hat{\rho}_2 - \hat{\rho}_1$. In the bottom panel, the vertical axis represents $\hat{\theta}_1$. Note that all $\hat{\theta}_2$s are not statistically significantly different from zero and all $\hat{\theta}_1$s are statistically significantly different from zero at the 5% significance level. The sensitive analysis results in Fig. 1 show that there is robust evidence of TVRRA.

Second, we carry out sensitivity analyses with respect to income variables. For example, we have tried to use head's labor income, wife's labor income, and household's income, as proxy of Y_t; and use the averages of head's labor income, wife's labor income, and household's income to divide the 1984–1999 subsample. In the majority of these cases, we have obtained similar results as we have shown in Sections 3.6.2 and 3.6.3.

Third, we also carry out sensitivity analyses using other specifications of Eqs. (3.1) and (3.2). Namely, we drop and add some variables in Eqs. (3.1) and (3.2) and obtain similar results as we have shown in Sections 3.6.2 and 3.6.3.

Finally, we use liquid wealth to do similar regressions as a additional check. Risky assets are liquid wealth minus risk-free assets, and risky share is the holdings of risky assets divided by liquid wealth. We use the same labor income in our empirical analysis of both liquid risky shares and financial risky shares. With liquid wealth, we obtain the following results. (1) Our estimates of the responses through the habit channel are not statistically significantly different from zero in the semi-strong form testing while the estimates are negative and statistically significantly different from zero in the strong form testing. In addition, our estimates of the responses through the income channel are not statistically significant. And (2) in the weak form testing, our second stage estimates of the responses through both channels are not statistically significant in both groups.[13] However, our empirical results about liquid wealth should be interpreted with caution when we use labor income as a proxy of the income corresponding to liquid wealth. This is simply because the relationship between liquid wealth and such a proxy is not in line with the relationship between wealth and income in that theoretical model.

3.6.5. Summary

Our empirical results about the responses through the income channel provide strong justification to our theoretical model. With such a model, our hypothesis essentially implies that controlling for the response through the income channel for each household and the heterogeneous responses through the habit channel across households will help generate a positive increase of ρ from the $i=1$ group to the $i=2$ group. In this regard, our robust and significant empirical results about the responses through the habit channel provide strong support for such a key hypothesis. Given all of these jointly estimated results, we argue, in terms of the testable theoretical predictions, that the weak form of TVRRAI is strongly supported by the PSID data.

4. Conclusion

In this paper, we introduce time-varying labor income, an empirically important element, into a portfolio choice model with external habit. The key theoretical contribution of our paper is that our analytical solution adds the following new insights to the literature. First, risky shares respond to wealth fluctuations through two channels, habit channel and labor income channel, and an internal misspecification problem arises if the labor income channel is ignored. Second, depending on whether they experience a large drop of income or not, households respond differently through the habit channel, and an external misspecification problem arises if the heterogeneous responses through the habit channel across households are ignored. Finally, if habit substantially deviates from its mean, a household whose relative risk aversion is decreasing in wealth may reduce its risky share after it becomes richer.

Also we test the semi-strong form and the weak form of TVRRAI. Our empirical contribution is that we find evidence of the weak form of TVRRAI. In this regard, our empirical results about the responses through the income channel provide justification to the use of our benchmark theoretical model; and out empirical results about the responses through the habit channel provide strong evidence of the weak form of TVRRAI in the household level data. Our refined results provide some confidence with respect to the use of habit formation preferences in macro-models. Even though our results reject the semi-strong form of TVRRAI, in line with the rejection of the strong form of TVRRAI in the literature, our acceptance of the weak form shows the importance of controlling for the internal and external misspecification problems. In addition, our analysis shows some potential of bridging the gap between the success of macro-models with habit and the previous negative evidence in micro-data by using more realistic theoretical models to identify the estimation.

Some questions still remain open, addressed here. First, the effect of inertia on portfolio adjustments remains unchanged from those in Brunnermeier and Nagel (2008), which casts reasonable doubt on the soundness of TVRRA. Thus, the strong asset allocation inertia identified in Brunnermeier and Nagel (2008) remains an interesting and not well-understood phenomenon. Second, the relation between risky shares and financial wealth in Eq. (2.3) is indeed highly nonlinear. Our estimates are based on linear regression models and they may be biased due to linear approximation. In a separate project, we will develop directly nonlinear/nonparametric estimates. By overcoming the bias associated with the linear estimates when the underlying relation is highly nonlinear, we will further explore what additional insights we can obtain about the time-varying relative risk aversion. Third, one limitation of our model is the abstraction of income shocks. How income shocks affect portfolio decision in a model with habit preference is an interesting issue that we might explore in the future.

[13] More results that are associated with liquid wealth are available upon request.

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Appendix A

A.1. The Derivation of Eq. (2.3)

To obtain a close-form solution in our benchmark model, we take the following strategy. We start with a model that extends Samuelson (1969) in which he derives analytical solutions, extend the model one place a time, and finally obtain our benchmark model. We impose additional restrictions such as Eqs. (2.1) and (2.2) in order to derive the analytical solution. In particular, we consider four different models (with the last one being our benchmark model).

For each of these four models, we take the following steps to derive its analytical solution:

1. Define the original model. Impose restrictions such as Eqs. (2.1) and (2.2) whenever necessary.
2. Choose an investment strategy.
3. Write the law of motion of wealth with the investment strategy.
4. Define new auxiliary variables.
5. Rewrite the law of motion of wealth with newly defined auxiliary variables.
6. Re-define a transformed model with auxiliary variables in such a way that the transformed model has an analytical solution.
7. Use the following relation to back out the solution to the risky share in the original model: the amount of money that is invested in the risky assets is equal to that invested in the risky assets in the transformed model.

A.1.1. Model I: adding fixed labor income and constant habit

In this subsection, we introduce a constant labor income flow and constant external habit into the model proposed by Samuelson (1969). This original model is denoted as Model I.

1. Define the original model.
   - The household chooses C_t and the share of $W_t - C_t + Y$ invested in the risky asset, α_t^1, to maximize

$$\max_{\{C_t,\,\alpha_t^1\}} \mathbb{E}[U(C_t, X, \delta, \gamma)] \quad \text{subject to } W_{t+1} = (1 + R_{p,t+1})(W_t - C_t + Y). \tag{A.1}$$

   where C_t denotes the consumption, Y denotes the labor income, W_t denotes the wealth, R_p denotes the portfolio return rate, and X the denotes external habit. The life-time utility function is defined as $U(C, X, \delta, \gamma) = \sum_{t=0}^{\infty} \delta^t \frac{(C-X)^{1-\gamma}}{1-\gamma}$.
2. Choose an investment strategy.
   - We divide the total household post-consumption wealth, $W_t + Y - C_t$ into two parts: $W_t + Y - C_t - (X - Y)/R_f$ and $(X - Y)/R_f$.
   - For the first part, the household invests a fraction $\tilde{\alpha}_t^1$ in the risky asset and the rest in the risk-free asset. The return to this (partial) wealth portfolio, $W_t + Y - C_t - (X - Y)/R_f$, is then given by $\tilde{R}_{p,t+1}^1 = \tilde{\alpha}_t^1 (R_t - R_f) + R_f$.
   - The second part, the remaining $(X - Y)/R_f$, is 100% invested in the risk-free asset.
   - As a result, the share of $W_t - C_t + Y$ invested in the risky asset is

$$\alpha_t^1 = \tilde{\alpha}_t^1 \times \frac{[W_t - C_t + Y - (X - Y)/R_f]}{(W_t - C_t + Y)}.$$

3. Write the law of motion of wealth with the investment strategy.
   - The law of motion of wealth is thus given by

$$W_{t+1} = \left(1 + \tilde{R}_{p,t+1}^1\right)(W_t - C_t + Y - (X - Y)/R_f) + (1 + R_f)(X - Y)/R_f.$$

4. Define new auxiliary variables.
   - We define two new auxiliary variables, $\tilde{W}_t^1$ and $\tilde{C}_t^1$:

$$\tilde{W}_t^1 = W_t - (X - Y) - (X - Y)/R_f,$$
$$\tilde{C}_t^1 = C_t - X.$$

5. Find the law of motion of wealth with newly defined auxiliary variables.
   - From the law of motion of W_{t+1}, we obtain

$$W_{t+1}-(1+R_f)(X-Y)/R_f=\left(1+\tilde{R}^1_{p,t+1}\right)\left(W_t-C_t+Y-(X-Y)/R_f\right).$$

   - By definition, we have

$$\tilde{W}^1_{t+1}=W_{t+1}-(1+R_f)(X-Y)/R_f,$$
$$\tilde{W}^1_t-\tilde{C}^1_t=\left(W_t-C_t+Y-(X-Y)/R_f\right).$$

   - Thus, the law of motion of wealth can be re-written as

$$\tilde{W}^1_{t+1}=\left(1+\tilde{R}^1_{p,t+1}\right)\left(\tilde{W}^1_t-\tilde{C}^1_t\right).$$

6. Re-define a transformed model with auxiliary variables in such a way that the transformed model has an analytical solution.
   - The model (Model I) defined in (A.1) can be transformed as

$$\max_{\{\tilde{C}^1_t,\tilde{\alpha}^1_t\}^\infty_{t=0}} \mathbb{E}\left[U(\tilde{C}^1_t,0,\delta,\gamma)\right] \quad \text{s.t. } \tilde{W}^1_{t+1}=\left(1+\tilde{R}^1_{p,t+1}\right)\left(\tilde{W}^1_t-\tilde{C}^1_t\right).$$

   - This transformed model is identical to the one studied in Samuelson (1969), which has a closed form solution, $\alpha^{*,\text{Samuelson}}$, under the condition that the expected returns and volatility are constant. In particular, $\alpha^{*,\text{Samuelson}}\approx 1$.
   - Thus, the optimal share of the risky asset, $\tilde{\alpha}^{1*}_t$, to the above transformed optimization problem is given by

$$\tilde{\alpha}^{1*}_t=\alpha^{*,\text{Samuelson}}\approx 1.$$

7. Use the following relation to back out the solution to the risky share in the original model: the amount of money that is invested in the risky assets is equal to that invested in the transformed model.

   The solution to the risky share in Model I is given by

$$\alpha^{1*}_t=\tilde{\alpha}^{1*}_t\times\frac{\left[W_t-C_t+Y-(X-Y)/R_f\right]}{(W_t-C_t+Y)}\approx 1-\frac{X-Y}{(W_t-C_t+Y)R_f}.$$

Note that we have used $\tilde{\alpha}^{1*}_t\approx 1$.

A.1.2. Model II: time-varying habit

In this subsection, we allow habit to be time-varying but set income $Y_t=0$. We define this model as Model II.

1. Define the original model.
   - Household's optimization problem is to maximize

$$\max_{\{C_t,\alpha^2_t\}^\infty_{t=0}} \mathbb{E}[U(C_t,\, X_t,\, \delta,\gamma)] \quad \text{s.t. } W_{t+1}=\left(1+R_{p,t+1}\right)(W_t-C_t), \tag{A.2}$$

   where X_t is the time-varying habit.
   - We impose the following restrictions on habit

$$(X_{t+1}-X)=\eta(X_t-X), \tag{A.3}$$

   where $|\eta|<1$. This is the same as Eq. (2.2). Even though our specification might be restrictive, such a process enables us to derive the close-form solution.
2. Choose an investment strategy.
   - We divide the total household post-consumption wealth, W_t-C_t, into two parts: $W_t-C_t-(X_t-X)/(Z+R_f)$ and $(X_t-X)/(Z+R_f)$, where $Z=(1+R_f)/\eta-(1+R_f)$.
   - In the first part, the household invests a fraction $\tilde{\alpha}^2_t$ in the risky asset and the rest in the risk-free asset. The return to this (partial) wealth portfolio is $\tilde{R}^2_{p,t+1}=\tilde{\alpha}^2_t(R_t-R_f)+R_f$.
   - The second part, the remaining $(X_t-X)/(Z+R_f)$, is 100% invested in the risk-free asset.
   - As a result, the share of W_t-C_t invested in the risky asset is

$$\alpha^2_t=\tilde{\alpha}^2_t\left[1-\frac{X_t-X}{(W_t-C_t)(Z+R_f)}\right] \tag{A.3}$$

3. Write the law of motion of wealth with the investment strategy.
   - The law of motion of wealth is thus given by

$$W_{t+1} = \left(1+\tilde{R}^2_{p,t+1}\right)(W_t - C_t - (X_t - X)/(Z+R_f)) + (1+R_f)(X_t - X)/(Z+R_f),$$
$$= \left(1+\tilde{R}^2_{p,t+1}\right)\{W_t - (X_t - X) - (X_t - X)/(Z+R_f) - [C_t - (X_t - X)]\} + (X_{t+1} - X)(Z+1+R_f)/(Z+R_f),$$

   where the last step uses Eq. (A.3) and the definition of Z.
4. Define new auxiliary variables.
   - We define two new auxiliary variables, $\tilde{W}^2_t$ and $\tilde{C}^2_t$:

$$\tilde{W}^2_t = W_t - (X_t - X) - (X_t - X)/(Z+R_f), \tilde{C}^2_t = C_t - X_t + X.$$

5. Find the law of motion of wealth with newly defined auxiliary variables.
   - We move $(X_{t+1} - X)(Z+1+R_f)/(Z+R_f)$ in the old law of motion from the right-hand-side to the left-hand-side and obtain

$$W_{t+1} - (X_{t+1} - X)(Z+1+R_f)/(Z+R_f) = \left(1+\tilde{R}^2_{p,t+1}\right)\{W_t - (X_t - X) - (X_t - X)/(Z+R_f) - [C_t - (X_t - X)]\}$$

   - By definition, we have

$$\tilde{W}^2_{t+1} = W_{t+1} - (X_{t+1} - X)(Z+1+R_f)/(Z+R_f),$$

$$\tilde{W}^2_t - \tilde{C}^2_t = \{W_t - (X_t - X) - (X_t - X)/(Z+R_f) - [C_t - (X_t - X)]\}.$$

   - Thus, the law of motion of wealth can be re-written as

$$\tilde{W}^2_{t+1} = \left(1+\tilde{R}^2_{p,t+1}\right)\left(\tilde{W}^2_t - \tilde{C}^2_t\right).$$

6. Re-define a transformed model with auxiliary variables in such a way that the transformed model has an analytical solution.
   - By definition we have $C_t - X_t = \tilde{C}^2_t - X$. Thus, Model II defined in (A.2) can be rewritten as the following transformed model

$$\max_{\{\tilde{C}^2_t,\, \tilde{\alpha}^2_t\}_{t=0}^{\infty}} = \mathbb{E}[U(\tilde{C}^2_t,\, X,\, \delta, \gamma)] \quad \text{s.t. } \tilde{W}^2_{t+1} = \left(1+\tilde{R}^2_{p,t+1}\right)\left(\tilde{W}^2_t - \tilde{C}^2_t\right),$$

   which is the same as the special case of Model I when $Y=0$.
   - The solution to the risky share in the transformed model is given by

$$\tilde{\alpha}^{2*} = \alpha^{1*}_t|_{Y=0} = \tilde{\alpha}^{1*}_t \times \frac{\tilde{W}^2_t - \tilde{C}^2_t - X/R_f}{\tilde{W}^2_t - \tilde{C}^2_t} \approx 1 - \frac{X}{\left(\tilde{W}^2_t - \tilde{C}^2_t\right)R_f} = 1 - \frac{X}{\left(W_t - C_t - \dfrac{X_t - X}{Z+R_f}\right)R_f} \tag{A.5}$$

   Note that $\tilde{\alpha}^{1*}_t \approx 1$; and in the last step, we have replaced $\tilde{W}^2_t$ and $\tilde{C}^2_t$ with their definitions.
7. Use the following relation to back out the solution to the risky share in the original model: the amount of money that is invested in the risky assets is equal to that invested in the transformed model.

The solution to the risky share in Model II is given by

$$\alpha^{2*}_t = \tilde{\alpha}^{2*}_t \left[1 - \frac{X_t - X}{(W_t - C_t)(Z+R_f)}\right] = \left[1 - \frac{X}{\left(W_t - C_t - \dfrac{X_t - X}{Z+R_f}\right)R_f}\right]\left[1 - \frac{X_t - X}{(W_t - C_t)(Z+R_f)}\right]. \tag{A.6}$$

A.1.3. Model III: time-varying labor income and fixed habit

In this subsection, we fix external habit but allow time-varying Y_t. We define this model as Model III.

1. Define the original model.

- Household's optimization problem is to maximize

$$\max_{\{C_t,\ \alpha_t^3\}_{t=0}^{\infty}} \mathbb{E}[U(C_t,\ X,\ \delta,\gamma)] \quad \text{s.t. } W_{t+1} = \left(1+R_{p,t+1}\right)(W_t+Y_t-C_t), \tag{A.7}$$

- We impose the following restrictions on income

$$(Y_{t+1}-Y) = \kappa(Y_t-Y) \tag{A.8}$$

  where $|\kappa| < 1$. This is the same as Eq. (2.1).

2. Choose an investment strategy.
   - The total household post-consumption wealth is given by $W_t+Y_t-C_t$.
   - A fraction $\tilde{\alpha}_t^3$ of $W_t+Y_t-C_t$ is invested in the risky assets and the rest in the risk-free asset. The return to this (partial) wealth portfolio is $\tilde{R}_{p,t+1}^3 = \tilde{\alpha}_t^3(R_t-R_f)+R_f$.
3. Write the law of motion of wealth with the investment strategy.
   - The law of motion of wealth is thus given by

$$W_{t+1} = \left(1+\tilde{R}_{p,t+1}^3\right)(W_t+Y_t-C_t).$$

4. Define new auxiliary variables.
   - We define four new auxiliary variables, $\tilde{W}_t^3$, $\tilde{C}_t^3$, $\tilde{X}_t^3$, and $\tilde{X}^3$:

$$\tilde{W}_t^3 = W_t,$$

$$\tilde{C}_t^3 = C_t-Y_t,$$

$$\tilde{X}_t^3 = X-Y_t,$$

$$\tilde{X}^3 = X-Y.$$

5. Find the law of motion of wealth with newly defined auxiliary variables.
   - With the two new auxiliary variables, $\tilde{W}_t^3$ and $\tilde{C}_t^3$, the law of motion of wealth can be re-written as

$$\tilde{W}_{t+1}^3 = \left(1+\tilde{R}_{p,t+1}^3\right)\left(\tilde{W}_t^3-\tilde{C}_t^3\right).$$

6. Re-define a transformed model with auxiliary variables in such a way that the transformed model has an analytical solution.
   - Note that by definition we have $C_t-X_t = \tilde{C}_t^3-\tilde{X}_t^3$. Thus, Model II defined in (A.7) can be rewritten as the following transformed model

$$\max_{\{\tilde{C}_t^3,\ \tilde{\alpha}_t^3\}_{t=0}^{\infty}} = \mathbb{E}[U(\tilde{C}_t^3,\ \tilde{X}_t^3,\ \delta,\gamma)] \quad \text{s.t. } \tilde{W}_{t+1}^3 = \left(1+\tilde{R}_{p,t+1}^3\right)\left(\tilde{W}_t^3-\tilde{C}_t^3\right),$$

   - In addition, it is straightforward to show that

$$\tilde{X}_{t+1}^3-\tilde{X}^3 = \kappa\left(\tilde{X}_t^3-\tilde{X}^3\right).$$

   - This transformed model is the same model as Model II and the law of motion of $\tilde{X}_t^{3*}$ is also the same as the one imposed in Model II.
   - Thus, the solution to the risky share in the transformed model is given by

$$\tilde{\alpha}_t^{3*} = \left[1-\frac{\tilde{X}^3}{\left(\tilde{W}_t^3-\tilde{C}_t^3-\dfrac{\tilde{X}_t^3-\tilde{X}^3}{Z+R_f}\right)R_f}\right]\left[1-\frac{\tilde{X}_t^3-\tilde{X}^3}{\left(\tilde{W}_t^3-\tilde{C}_t^3\right)(Z+R_f)}\right].$$

7. Use the following relation to back out the solution to the risky share in the original model: the amount of money that is invested in the risky assets is equal to that invested in the transformed model.

- Since the original model and the transformed model should have the same solution to the amount of wealth invested in risky assets, we have

$$\alpha_t^{3*}(W_t - C_t + Y_t) = \tilde{\alpha}_t^{3*}\left(\tilde{W}_t^3 - \tilde{C}_t^3\right)$$

- By definition, we have

$$\left(\tilde{W}_t^3 - \tilde{C}_t^3\right) = (W_t - C_t + Y_t).$$

- Thus, we have

$$\alpha_t^{3*} = \tilde{\alpha}_t^{3*} = \left[1 - \frac{\tilde{X}^3}{\left(\tilde{W}_t^3 - \tilde{C}_t^3 - \frac{\tilde{X}_t^3 - \tilde{X}^3}{Z + R_f}\right) R_f}\right]\left[1 - \frac{\tilde{X}_t^3 - \tilde{X}^3}{\left(\tilde{W}_t^{3*} - \tilde{C}_t^{3*}\right)(Z + R_f)}\right]$$
$$= \left[1 - \frac{X - Y}{\left(W_t - C_t + Y_t + \frac{Y_t - Y}{Z + R_f}\right) R_f}\right]\left[1 + \frac{Y_t - Y}{(W_t - C_t + Y_t)(Z + R_f)}\right]. \tag{A.9}$$

A.1.4. Model IV (The Benchmark Model): Time-varying labor income and time-varying habit

In this subsection, we allow both time-varying external habit and time-varying Y_t. For such a case, we define this model as Model IV, which is also the benchmark model in our paper.

1. Define the original model.
   - Household's optimization problem is to maximize

$$\max_{\{C_t,\ \alpha_t\}_{t=0}^{\infty}} \mathbb{E}[U(C_t,\ X_t,\ \delta, \gamma)] \quad \text{s.t. } W_{t+1} = \left(1 + R_{p,t+1}\right)(W_t + Y_t - C_t), \tag{A.10}$$

   - We impose the following restrictions on income

$$(Y_{t+1} - Y) = \kappa(Y_t - Y) \tag{A.11}$$

$$(X_{t+1} - X) = \eta(X_t - X), \tag{A.12}$$

   where $|\kappa| < 1$ and $|\eta| < 1$. These are the same as Eqs. (2.1) and (2.2).
2. Choose an investment strategy.
   - The total household post-consumption wealth is given by $W_t + Y_t - C_t$.
   - A fraction α_t of $W_t + Y_t - C_t$ is invested in the risky assets and the rest in the risk-free asset. The return to this (partial) wealth portfolio is $\tilde{R}_{p,t+1} = \alpha_t\left(R_t - R_f\right) + R_f$.
3. Write the law of motion of wealth with the investment strategy.
   - The law of motion of wealth is thus given by

$$W_{t+1} = \left(1 + \tilde{R}_{p,t+1}\right)(W_t + Y_t - C_t).$$

4. Define new auxiliary variables.
   - We define four new auxiliary variables, $\tilde{W}_t$, $\tilde{C}_t$, $\tilde{X}_t$, and $\tilde{X}$:

$$\tilde{W}_t = W_t,$$
$$\tilde{C}_t = C_t - Y_t,$$
$$\tilde{X}_t = X_t - Y_t,$$
$$\tilde{X} = X - Y.$$

5. Find the law of motion of wealth with newly defined auxiliary variables.
   - With the two new auxiliary variables, $\tilde{W}_t$ and $\tilde{C}_t$, the law of motion of wealth can be re-written as

$$\tilde{W}_{t+1} = \left(1 + \tilde{R}_{p,t+1}\right)\left(\tilde{W}_t - \tilde{C}_t\right).$$

6. Re-define a transformed model with auxiliary variables in such a way that the transformed model has an analytical solution.
   - Note that by definition we have $C_t - X_t = \tilde{C}_t - \tilde{X}_t$. Thus, Model II defined in (A.10) can be rewritten as the following transformed model

$$\max_{\{\tilde{C}_t,\, \tilde{\alpha}_t\}_{t=0}^{\infty}} = \mathbb{E}[U(\tilde{C}_t,\, \tilde{X}_t,\, \delta, \gamma)] \quad \text{s.t. } \tilde{W}_{t+1} = \left(1+\tilde{R}_{p,t+1}\right)\left(\tilde{W}_t - \tilde{C}_t\right),$$

   - In addition, we can show that

$$\tilde{X}_{t+1} - \tilde{X} = \kappa\left(\tilde{X}_t - \tilde{X}\right). \tag{3.13}$$

     - We use the definitions of $\tilde{X}_{t+1}$, $\tilde{X}$, and $\tilde{X}_t$ into the above equation and obtain

$$\tilde{X}_{t+1} - \tilde{X} = (X_{t+1} - Y_{t+1}) - (X - Y) = \kappa[(X_t - Y_t) - (X - Y))] = \kappa\left(\tilde{X}_t - \tilde{X}\right).$$

   - This transformed model is the same model as Model II and the law of motion of $\tilde{X}_t$ is also the same as the one imposed in Model II.
   - Thus, under the condition that expected return and the standard deviation of R_t are constant, the solution to the risky share in the transformed model is given by

$$\tilde{\alpha}_t^* = \left[1 - \frac{\tilde{X}}{\left(\tilde{W}_t - \tilde{C}_t - \frac{\tilde{X}_t - \tilde{X}}{Z+R_f}\right)R_f}\right]\left[1 - \frac{\tilde{X}_t - \tilde{X}}{\left(\tilde{W}_t - \tilde{C}_t\right)(Z+R_f)}\right],$$

   where Z is defined as $Z = \left(\frac{1}{\kappa} - 1\right)(1+R_f)$.
7. Use the following relation to back out the solution to the risky share in the original model: the amount of money that is invested in the risky assets is equal to that invested in the transformed model.
   - Since the original model and the transformed model should have the same solution to the amount of wealth invested in risky assets, we have

$$\alpha_t^*(W_t - C_t + Y_t) = \tilde{\alpha}_t^*\left(\tilde{W}_t - \tilde{C}_t\right)$$

   - By definition, we have

$$\left(\tilde{W}_t - \tilde{C}_t\right) = (W_t - C_t + Y_t).$$

   - Thus, we have

$$\begin{aligned}\alpha_t^* = \tilde{\alpha}_t^* &= \left[1 - \frac{\tilde{X}}{\left(\tilde{W}_t - \tilde{C}_t - \frac{\tilde{X}_t - \tilde{X}}{Z+R_f}\right)R_f}\right]\left[1 - \frac{\tilde{X}_t - \tilde{X}}{\left(\tilde{W}_t - \tilde{C}_t\right)(Z+R_f)}\right] \\ &= \left[1 - \frac{X-Y}{\left(W_t - C_t + Y_t + \frac{(Y_t - Y) - (X_t - X)}{Z+R_f}\right)R_f}\right]\left[1 + \frac{(Y_t - Y) - (X_t - X)}{(W_t - C_t + Y_t)(Z+R_f)}\right].\end{aligned} \tag{A.14}$$

   - Eq. (A.14) is Eq. (2.3) and it provides an analytical solution that enables us to discuss how the risky share responds to post-consumption wealth and how time-varying labor income and time-varying external habit affect the response.

A.2. The derivation of Eq. (2.6)

Define $\tilde{W}_t = W_t - C_t$ and $w_t = \log(\tilde{W}_t)$, we can rewrite Eq. (2.5) as

$$\alpha_t = 1 - \frac{X-Y}{(W_t - C_t + Y)R_f} \equiv 1 - f(w_t),$$

where $f(w_t) = \frac{X-Y}{(e^{w_t}+Y)R_f}$. We approximate $f(w_t)$ around a point, w the mean of wealth w_t, up to the first order, and then obtain

$$f(w_t) \approx f(w) + \left[\frac{df(w_t)}{dw_t}\bigg|_{w_t = w}\right](w_t - w) = f(w) + \frac{(X-Y)e^w}{(e^w+Y)^2 R_f} w - \frac{(X-Y)e^w}{(e^w+Y)^2 R_f} w_t \equiv f(w) + \Theta w - \Theta w_t,$$

where $\Theta = \frac{(X-Y)e^w}{(e^w+Y)^2 R_f}$. Note the values of $f(w)$, Θ, and w do not depend on time. Thus,

$$\alpha_t \approx 1 - \left[f(w) + \Theta w - \Theta w_t\right] = \left[1 - f(w) - \Theta w\right] + \Theta w_t.$$

By taking the first-order difference (over time), we get

$$\Delta\alpha_t \approx \Theta \Delta w_t = \frac{Xe^w}{(e^w+Y)^2 R_f}\Delta w_t - \frac{Ye^w}{(e^w+Y)^2 R_f}\Delta w_t = \rho\Delta w_t + \theta Y \Delta w_t,$$

where $\rho = \frac{Xe^w}{(e^w+Y)^2 R_f}$ and $\theta = \frac{-e^w}{(e^w+Y)^2 R_f}$. This finishes the proof of Eq. (2.6).

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