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Time-varying relative risk aversion: Theoretical mechanism and empirical evidence☆

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ABSTRACT

This paper explores the issue of understanding time-varying relative risk aversion with household-level data on two classical portfolio choice problems. First, we derive an analytic form solution to a parsimonious portfolio choice model with the preference given by Greenwood, Hercowitz and Huffman (1988, GHH), and then, the solution identifies four partial equilibrium effects in our model with the GHH preference on risky shares through two channels and two net effects whose signs hinge on the value of a key structural parameter. Based on household-level data, our empirical results from both mean and quantile regression models show clearly that wealth negatively affects risky shares and the estimated effects are statistically significant and robust, which is in line with the theory. Finally, we show that the GHH preference alone is not sufficient in explaining how risky shares respond to labor income in the household-level data.

1. Introduction

This paper tries to understand time-varying relative risk aversion (hereafter TVRRA) with household data on two important portfolio choice problems, and how financial wealth and labor income affect risky shares. In particular, this paper provides an underlying theory of TVRRA and documents robust empirical evidence. TVRRA has been widely regarded as a key behind the success of theoretical models in explaining various important economic phenomena about macro-data. For example, models with habit formation preferences match stylized facts about asset returns and business cycle moments, see, for instance, Constantinides (1990), Jermann (1998), among many others. These models use habit formation preferences to generate TVRRA. However, our understanding of TVRRA with micro-data is unsatisfactory. The results from recent studies cast doubt on the use of habit formation preferences to generate TVRRA as the underlying mechanism that helps understand micro financial data. Specifically, Brunnermeier and Nagel (2008) showed that the unconditional predictions due to habit formation preferences on portfolio choices are rejected by

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the Panel Study of Income Dynamics (hereafter PSID) data from 1984 to 2003. More importantly, existing works have not provided possible underlying mechanisms in generating TVRRA that could explain portfolio choice facts in the household level data.

In this paper, we analyze whether TVRRA generated by the preference proposed in Greenwood, Hercowitz and Huffman (1988, hereafter GHH) can provide a plausible underlying mechanism such that the generated TVRRA helps understand key facts about household-level portfolio choice data. For this purpose, we explore both theoretical implications of our portfolio choice model with the GHH preference and empirical evidence in the PSID data on two classical questions: how risky shares respond to financial wealth and how they respond to labor income. To derive theoretical implications, we solve a simple discrete-time portfolio choice model with the GHH preference and obtain theoretical predictions about the relationships among risky shares, financial wealth, and labor income.[1] Our choice of the GHH preference is fundamentally different from that in both (Brunnermeier and Nagel, 2008) and Liu et al. (2016), which assumed habit formation preferences. To examine empirical evidence, we conduct extensive tests with two-stage least squares (2SLS) estimations and quantile analyses with the PSID data from 1984 to 2015, while both (Brunnermeier and Nagel, 2008) and Liu et al. (2016) used much shorter periods of the PSID data and did not do quantile estimations. In this paper, we focus on addressing the two classical questions from the above, but both Brunnermeier and Nagel (2008) and Liu et al. (2016) only explored the first question, how risky shares respond to financial wealth.

Main contributions of this paper to the literature can be described as follows. First, we derive a closed-form theoretical solution to risky shares in our parsimonious model under simplified assumptions and then provide new insights on how households make their portfolio choice decisions. Specifically, we show that, with the GHH preference, both financial wealth and labor income affect risky shares through two channels, the labor channel and the leisure channel. Through the labor channel, there are two partial equilibrium effects. The first partial equilibrium effect implies that, through the labor channel, households become less aggressive in adjusting their risky shares in response to financial wealth accumulations. The effect is the labor income relaxation effect on the relationship between risky shares and financial wealth, an effect discussed in Liu et al. (2016).[2] The second partial equilibrium effect implies that, through the labor channel, households are more aggressive in their portfolio choices when labor income is larger. This is the labor income insurance effect on the relationship between labor income and risky shares, an effect that has been initially and formally defined in Bodie et al. (1992).[3]

Through the leisure channel, our model with the GHH preference engenders two opposite partial equilibrium effects (with respect to the above two effects through the labor channel) on risky shares. The third partial equilibrium effect is about how financial wealth affects risky shares through the leisure channel. In particular, this effect implies that, with positive labor effort thus disutility, households become more aggressive in adjusting their risky shares in response to financial wealth accumulations. This is a leisure tightening effect on the relationship between risky shares and financial wealth, an effect that is analogous to the more general ones discussed in Stiglitz (1969). The fourth partial equilibrium effect is about how labor income affects risky shares through the leisure channel. It implies that households are less aggressive in their portfolio choices when their labor income is higher. The last effect is the leisure de-insurance effect, which is opposite to the labor income insurance effect, and a new addition to the literature.

Our second highlighted theoretical finding is that, with the GHH preference, both the net effect of financial wealth on risky shares and that of labor income on risky shares are driven by one key structural parameter: the sensitivity of labor supply to real wage rates, a parameter that determines the wage elasticity of labor supply. Thus, we provide important new theoretical results and insights to the literature by showing these effects and presenting them in one analytical solution with one parameter.

Third, we make important empirical contributions by documenting empirical regularities in the PSID data. We provide robust empirical evidence of how financial wealth affects portfolio choices. Our regression results show that risky shares respond negatively to financial wealth accumulations in the data. The results are statistically significant and robust for both subsamples (details about the two subsamples are shown in Section 3.1) with and without outliers. Interestingly, they also hold across various quantiles of risky shares when the density distribution of risky shares contains fat tails.[4]

Our empirical results show that the overall responses of risky shares to labor income are statistically insignificant and robust for both subsamples with and without outliers. They also hold across various quantiles of risky shares when the density distribution of risky shares contains fat tails. Even though they are not qualitatively different from the existing findings in the literature [see Guiso et al. (1996), Heaton and Lucas (2000), among others], our contribution is that we extend this empirical regularity into the PSID data from 1984 to 2015.

Finally, our findings shed new insights on the understanding of TVRRA in fitting micro financial data. The empirical regularity that risky shares respond negatively to financial wealth accumulations is clear and there is strong evidence of TVRRA in the portfolio choice activities at the household level (in the PSID data from 1984 to 2015). This empirical regularity reaffirms the theoretical prediction of our model with the GHH preference within the standard range of the key parameter. In other words, we have successfully shown that our model with the GHH preference (thus automatically generating TVRRA) provides a plausible mechanism in understanding one important portfolio choice fact. Equally important, the empirical regularity that risky shares do not respond to the change of labor income deviates from the corresponding theoretical prediction. This conflict shows that TVRRA generated by the GHH preference is inadequate in characterizing how risky shares respond to labor income in the data. This open question should be addressed in a future research topic.

The rest of the paper is organized as follows. Section 2 lays out the theoretical model, obtains the analytical solution, and provides theoretical discussions. The main empirical analyses are conducted in Section 3, together with discussing how our findings shed light on understanding time-varying relative risk aversion in fitting micro-data. Our main conclusions are provided in Section 4.

1 With the GHH preference, we introduce labor supply into the model.

2 Liu et al. (2016) derived this effect with habit-formation preferences.

3 For the meaning of two channels, partial equilibrium effects, and net effects, please see Section 2.4.1 for details.

4 For this case, we run quantile regressions with instrumental variables.

2. Theoretical model

2.1. Model setup

A representative household lives infinitely, who decides consumption, labor effort, and portfolio choices (by investing in two assets: a risky asset and a risk-free asset) in each period. Formally, the household chooses consumption C_t, labor effort H_t, the risky asset holding position S_{t+1}, and risk-free asset holding position B_t to maximize its life-time utility:

$$U = \mathbb{E} \sum_{t=0}^{\infty} \delta^t \frac{\left(C_t - H_t^{\omega}/\omega\right)^{1-\sigma}}{1-\sigma}$$

subject to the period budget constraint as $C_t + B_t + S_{t+1} = (1 + R_t)S_t + (1 + R_f)B_{t-1} + z_t H_t$, where $\mathbb{E}$ denotes the unconditional expectation operator, δ represents the subjective discount factor, σ denotes the risk aversion coefficient, ω is a parameter related to the wage elasticity of labor supply (see Section 2.4.2), R_t stands for the rate of return of holding the risky asset from period $t-1$ to period t, R_f denotes the risk-free rate, z_t is total factor productivity, and $z_t H_t$ denotes the labor income of the household. Given the presence of labor income in our model, wealth in the model rightfully corresponds to financial wealth.

The utility function with the GHH preference was first introduced into the business cycle model in Greenwood et al. (1988). In addition to the failure of habit formation preferences in explaining risky shares in the household-level data as in Brunnermeier and Nagel (2008), three more reasons have been provided to justify the use of the GHH preference. First, the GHH preference neutralizes the wealth effect on labor supply, and labor input is determined independent of the intertemporal consumption and saving choices. As a result, the model in Greenwood et al. (1988) generates co-movements among consumption, labor input, and labor productivity when responding to the shocks on the marginal efficiency of investment. These hypotheses are generally supported by empirical evidence. In contrast, models using other preferences usually fail to do so, when the intertemporal consumption and saving choices affect labor supply as addressed by Barro and King (1984). The GHH preference has then been widely adopted to study various economic issues and these applications have been overwhelmingly successful. For example, models with the GHH preference have effectively explained dynamics of aggregate data [see Mendoza (1991), Garcia-Cicco et al. (2010), among others]. Second, empirical studies have shown that employment (i.e., labor supply at the extensive margin) is independent of financial wealth, see, for example, Chang and Kim (2007), Ferriere and Navarro (2018), and Yum (2018) for details. This empirical finding is in line with the GHH preference. Third, Jahan-Parvar et al. (2013) showed that the financial friction models with the GHH preference are able to explain the observed business cycles and equity premium in emerging economies. Nevertheless, it is still unclear whether the GHH preference can fit micro-data, in particular, related to portfolio choices.

2.2. Timeline and evolution of financial wealth

For the purpose of obtaining the analytical solution, it is important to know the timeline of the model and how financial wealth evolves over time:

- At the beginning of period t, the household inherits financial wealth, W_t.
- Within period t, the household makes decisions about consumption and labor effort. In particular, the household spends C_t on consumption and receives labor income $Y_t = z_t H_t$.
- At the end of period t, the household's wealth portfolio is given by $W_t - C_t + Y_t$ and the household determines the fraction of this wealth portfolio, $\alpha_t = S_{t+1}/(S_{t+1} + B_t)$, that is invested in the risky asset.
- The rate of return to this portfolio, $R_{p,t+1}$, is given by $R_{p,t+1} = \alpha_t(R_{t+1} - R_f) + R_f$.
- Thus, the financial wealth in the beginning of period $t+1$, W_{t+1}, is given by $W_{t+1} = \left(1 + R_{p,t+1}\right)\left(W_t - C_t + Y_t\right)$, which is the product of the gross rate of return, $1 + R_{p,t+1}$, and the financial wealth at the end of period t, $W_t - C_t + Y_t$.

2.3. Closed-form solution

One feature of the GHH preference is that labor effort is determined by independence of the intertemporal consumption and saving choices. Because of this feature, we take two parts to derive the solution to α_t. Specifically, we decompose the derivation of solving α_t into two sub-questions as follows. The first sub-question is to determine H_t and Y_t in the equilibrium. It is straightforward to show that in the equilibrium, we have

$$H_t = z_t^{1/(\omega-1)} \quad \text{and} \quad Y_t = z_t H_t = z_t^{\omega/(\omega-1)} = H_t^{\omega},$$

where both H_t and Y_t are solely determined by z_t. The second sub-question is to determine α_t for the given H_t and Y_t in the equilibrium. We follow the same logic as in Liu et al. (2016) to partially solve the portfolio choice problem here, i.e., to obtain a closed-form solution to α_t. For this purpose, we impose simplified assumptions, manipulate the objective function and financial wealth (in the beginning of period $t+1$), transform our portfolio choice problem into one that has the analytical solution under the simplified assumptions, and then back up the solution to α_t in the original portfolio choice problem accordingly. Following the above logic, we obtain the solution to α_t as:

$$\alpha_t^* = \alpha^S \left[1 - \frac{X}{\left(W_t - C_t + Y_t - \frac{X_t - X}{Z + R_f} \right) R_f} \right] \left[1 - \frac{X_t - X}{\left(W_t - C_t + Y_t \right)(Z + R_f)} \right]. \tag{1}$$

Here, $X_t = H_t^\omega/\omega - Y_t = \frac{1-\omega}{\omega} Y_t = \frac{(1-\omega)}{\omega} z_t^{\omega/(\omega-1)}$, $X = H^\omega/\omega - Y = \frac{1-\omega}{\omega} z^{\omega/(\omega-1)}$, and $Z = (1 + R_f)/\kappa - (1 + R_f)$. Throughout the paper, a variable without the time-subscript denotes the non-stochastic steady state of this variable: for example, X denotes the non-stochastic steady state of X_t. Besides, α^S is the solution to α as in Samuelson (1969) and its value is very close to 1. The next is to derive (1). To this end, the following four steps are taken to obtain the closed-form solution to α_t.

1. We impose the following two simplified conditions:
   (1) We assume that X_t follows an AR(1) process, such as $X_t - X = \kappa \left(X_{t-1} - X \right)$, where κ is a constant. This AR(1) process may be regarded as the first order linear approximation of the true data generating process of X_t. This assumption is important because it allows us to transform our portfolio choice model into a model with an analytical solution to risky shares.
   (2) We assume that the expected return and the standard deviation are constant, an assumption that is also imposed in Samuelson (1969) and Brunnermeier and Nagel (2008). This assumption is important because it allows the portfolio choice models in Samuelson (1969) and Brunnermeier and Nagel (2008) to have an analytical solution.
2. Let W_t denote the wealth portfolio in the beginning of period t. The wealth portfolio at the end of period t is given by $W_t - C_t + Y_t$. Suppose that the household invests a fraction α_t of this period-end wealth portfolio in the risky assets and the rest in the risk-free asset, and the return rate to this wealth portfolio, $R_{p,t+1}$, is $R_{p,t+1} = \alpha_t \left(R_{t+1} - R_f \right) + R_f$. The portfolio choice problem is to choose C_t and α_t to maximize

   $$U = \mathbb{E} \sum_{t=0}^{\infty} \delta^t \frac{\left(C_t - Y_t/\omega \right)^{1-\sigma}}{1-\sigma}$$

   subject to the law of motion of wealth portfolio as $W_{t+1} = \left(1 + R_{p,t+1} \right) \left(W_t + Y_t - C_t \right)$, where W_{t+1} denotes the wealth portfolio at the beginning of period $t+1$. Here, $Y_t = H_t^\omega$ is used.
3. Define $\tilde{C}_t = C_t - Y_t$. The portfolio choice problem is reduced to selecting $\tilde{C}_t$ and α_t to maximize

   $$U = \mathbb{E} \sum_{t=0}^{\infty} \delta^t \frac{\left(C_t - Y_t - \frac{(1-\omega)}{\omega} Y_t \right)^{1-\sigma}}{1-\sigma} = \mathbb{E} \sum_{t=0}^{\infty} \delta^t \frac{\left(\tilde{C}_t - \frac{(1-\omega)}{\omega} Y_t \right)^{1-\sigma}}{1-\sigma}$$

   subject to the law of motion of wealth portfolio as $W_{t+1} = \left(1 + R_{p,t+1} \right) \left(W_t - \tilde{C}_t \right)$.
4. Define $X_t = \frac{(1-\omega)}{\omega} Y_t$. The portfolio choice problem is to choose $\tilde{C}_t$ and α_t to maximize

   $$U = \mathbb{E} \sum_{t=0}^{\infty} \delta^t \frac{\left(\tilde{C}_t - X_t \right)^{1-\sigma}}{1-\sigma},$$

   subject to the law of motion of wealth portfolio as $W_{t+1} = \left(1 + R_{p,t+1} \right) \left(W_t - \tilde{C}_t \right)$. This portfolio choice problem is now exactly the same portfolio choice problem as discussed in Liu et al. (2016) (see pages 244–245).[5] Thus, under the two simplified assumptions, the solution to α_t is given by:

   $$\alpha_t^* = \alpha^S \left[1 - \frac{X}{\left(W_t - \tilde{C}_t - \frac{X_t - X}{Z + R_f} \right) R_f} \right] \left[1 - \frac{X_t - X}{\left(W_t - \tilde{C}_t \right)(Z + R_f)} \right],$$

   where α^S is the solution to risky shares in Samuelson (1969). We replace $\tilde{C}_t$ with $C_t - Y_t$ in the above solution and obtain (1).

Clearly, (1) allows us to discuss how the GHH preference affects risky shares with respect to their responses to financial wealth and labor income.

[5] To see the roles of the two simplified assumptions, the reader is referred to the paper by Liu et al. (2016).

2.4. Theoretical implications

In this section, we discuss the theoretical implications due to the GHH preference on risky shares. Our focus is on the static relationships among variables of interest in the simplest case in which the total factor productivity is a constant, $z_t \equiv z$. For this simplest case, we have $X_t \equiv X = H^{\omega}/\omega - Y$ and α_t^* is simplified as

$$\alpha^* = \alpha^S \left[\underbrace{1 + \frac{Y}{(W - C + Y) R_f}}_{\text{Labor channel}} \underbrace{- \frac{H^{\omega}/\omega}{(W - C + Y) R_f}}_{\text{Leisure channel}} \right]. \tag{2}$$

As decomposed in (2), we label two channels, the labor channel and the leisure channel. Note that there are two channels because our model assumes the GHH preference. Next, we present four partial equilibrium effects and two net effects about how financial wealth, W, and labor income, Y, affect risky shares, respectively.

2.4.1. Four partial equilibrium effects

In view of (2), both financial wealth, W, and labor income, Y, affect risky shares through the two channels. Thus, there are four partial equilibrium effects. The term partial equilibrium is used here because we do hold the ceteris paribus assumption and we have not used the equilibrium condition $Y = zH$.

Labor channel

To see how the two partial equilibrium effects on risky shares through the labor channel, we write the solution to risky shares associated with the channel as:

$$\alpha_Y^* = \alpha^S \left[1 + \frac{Y}{(W - C + Y) R_f} \right],$$

from which, it then follows that our model with the GHH preference has two effects on risky shares through the labor channel. The first effect is about the impact of financial wealth on risky shares through this channel. With positive labor income, households become less aggressive (more relaxing) in their portfolio choices when their financial wealth accumulates. Mathematically, this effect implies that $\partial \alpha_Y^* / \partial W < 0$ when $Y > 0$. One argument behind this effect is as follows. Everything else being equal, the occurrence of constant labor income automatically increases the size of risky risk-free assets and lower risky shares before the household re-balances its portfolio. We define this effect as the labor income relaxation effect. This is like the effect discussed in Liu et al. (2016). Note that we obtain this effect in our model with the GHH preference while Liu et al. (2016) obtained their effect with habit-formation preferences.

The second effect is about the impact of labor income on risky shares through the labor channel. This effect means that the ability to earn labor income incentivizes a household to assume greater risks in her investment portfolio. For example, according to this effect, we have $\alpha_Y^* > \alpha^S$ because $Y > 0$ (note that α^S is associated with the case in which labor income is zero). For another example, this effect implies that higher labor income may induce a household to assume greater risks in her investment portfolio (for the given financial wealth), i.e., $\partial \alpha_Y^* / \partial Y > 0$ when $Y > 0$ for the given financial wealth. We define this effect as the labor income insurance effect. Similar effect has been discussed in Bodie et al. (1992). According to Bodie et al. (1992), labor income provides an insurance to the household against adverse investment outcomes, and households, therefore, become more aggressive in their portfolio choices with a higher labor income for the given financial wealth.

Leisure channel

To see how the two partial equilibrium effects on risky shares through the leisure channel, we write the solution to risky shares associated with the channel as:

$$\alpha_H^* = \alpha^S \left[\frac{-H^{\omega}/\omega}{(W - C + Y) R_f} \right],$$

by which, one can see that our model with the GHH preference has two effects on risky shares through the leisure channel, which are qualitatively opposite of the two effects through the labor channel. The first effect is about the impact of financial wealth on risky shares through the leisure channel. With positive labor effort (i.e., $H > 0$) thus disutility, households become more aggressive (less relaxing) in their portfolio choices, i.e., increase their risky shares, when their financial wealth accumulates. Mathematically, this effect means that $\partial \alpha_H^* / \partial W > 0$ when $H > 0$. We label this effect as the leisure tightening effect. This effect is quite general. Indeed, Stiglitz (1969) showed that an important implication of non-homothetic utility is to reduce relative risk aversion. The GHH preference is non-homothetic. Thus, it is not surprising that households with the GHH preference increase their risky shares when their financial wealth accumulates through the leisure channel. This effect is similar to the effect that is due to the existence of habits as discussed in both Brunnermeier and Nagel (2008) and Liu et al. (2016). Our leisure tightening effect with the GHH preference is a new addition to the literature.

The second effect is about the impact of labor income on risky shares through the leisure channel. When households provide labor effort (i.e., $H > 0$) thus disutility, they become less aggressive in their portfolio choices when their labor income increases. In other words, the desire to have leisure induces the individual to assume less risks in her investment portfolio in responding to the increase of labor income. Mathematically, we have $\partial \alpha_H^* / \partial Y < 0$ when $H > 0$. This result is intuitive as labor and leisure are substitutes to each other. We define this effect as the leisure de-insurance effect. This effect is a new addition to the literature even though it may be regarded as a straightforward extension of the labor income insurance effect.

2.4.2. Two net effects

In Section 2.4.1, we have shown four partial equilibrium effects (through two channels) in our model with the GHH preference, two about the impact of financial wealth on risky shares and two about the impact of labor income on risky shares. In this section, we derive the net effect, in our model with the GHH preference, of financial wealth (labor income) on α^* using the two partial equilibrium effects related to financial wealth (labor income). Formally,

$$\alpha^* = \alpha^S \left[1 + \frac{Y - \frac{1}{\omega}(H)^\omega}{(W - C + Y) R_f} \right] = \alpha^S \left[1 + \frac{\left(1 - \frac{1}{\omega}\right) Y}{(W - C + Y) R_f} \right]. \tag{3}$$

The term net is used here and the equilibrium equality, $Y = zH$, is employed in the second equality, where $Y = z^{\frac{\omega}{\omega-1}}$ and $H = z^{\frac{1}{\omega-1}}$.

Net effect of financial wealth on risky shares

As discussed in Section 2.4.1, the labor income relaxation effect implies that for a given positive labor income, households decrease their risky shares when they accumulate financial wealth, i.e., $\partial\alpha^*_Y/\partial W < 0$ when $Y > 0$. The leisure tightening effect implies that with positive labor effort, households increase their risky shares when their financial wealth increases, i.e., $\partial\alpha^*_H/\partial W > 0$ when $H > 0$. Again, since $Y = zH$, the net effect of financial wealth on risky shares depends on the sizes and the weights of the two effects. An application of (3) leads to

$$\partial\alpha^*/\partial W = -(1 - 1/\omega) \times P, \tag{4}$$

where $P = \alpha^S Y / \left[(W - C + Y)^2 R_f\right]$ and is positive, which implies that whether $\partial\alpha^*/\partial W$ is positive, zero, or negative crucially depends on the value of ω. When financial wealth, W, increases, one can see from (4) that risky shares α^* decrease if $\omega > 1$, increase if $\omega < 1$, and do not change if $\omega = 1$. The exact value of ω is not known and it is typically assumed that $\omega > 1$ in the literature [see Mendoza (1991), Schmitt-Grohé and Uribe (2003), Neumeyer and Perri (2005), among many others]. Such a calibration is in line with the empirical estimates of the wage elasticity of labor supply, η, in the literature. Table 1 in Chetty (2006) showed that the estimates of η range from 0.033 to 1.040. Given that $\omega = \frac{1}{\eta} + 1$, it is reasonable to argue that empirical estimates of η in the literature imply that $\omega > 1$. Thus, (4) means that if households have GHH preference, they will decrease their risky shares when their financial wealth increases. Such an inverse relationship between risky shares and financial wealth is the first important theoretical prediction from the GHH preference.

Net effect of labor income on risky shares

As argued in Section 2.4.1, the labor income insurance effect implies that for a given positive level of financial wealth, households increase their risky shares as their labor income increases, i.e., $\partial\alpha^*_Y/\partial Y > 0$ when $Y > 0$. The leisure de-insurance effect implies that for the given positive financial wealth, households decrease their risky shares as their labor income increases, i.e., $\partial\alpha^*_H/\partial Y < 0$ when $H > 0$. Since $Y = zH$, the net effect of labor income on risky shares depends on the size and the weight of each effect. Therefore, (3) implies the following

$$\partial\alpha^*/\partial Y = (1 - 1/\omega) \times Q, \tag{5}$$

where $Q = \alpha^S (W - C) / \left[(W - C + Y)^2 R_f\right]$ and is positive.[6] Clearly, (5) indicates that whether $\partial\alpha^*/\partial Y$ is positive, zero, or negative crucially depends on the value of ω. In particular, when labor income Y increases, risky shares α^* increase if $\omega > 1$, decrease if $\omega < 1$, and remain unchanged if $\omega = 1$. Again, the empirical estimates of η in the literature imply that $\omega > 1$. Thus, (5) means that if households have the GHH preference, they will increase their risky shares when their labor income increases. The positive correlation between risky shares and labor income is the second important theoretical prediction from the GHH preference.

2.4.3. In relation to the literature

First, our discussions shed light on the classical question about how financial wealth theoretically affects risky shares. The literature suggests that for households with constant relative risk aversion preferences, their risky shares will not change as financial wealth, W, increases [see Samuelson (1969)], and if households have habit formation preferences, risky shares will increase with wealth [see Brunnermeier and Nagel (2008)]. Liu et al. (2016) introduced labor income into Brunnermeier and Nagel (2008)'s model (still using habit formation preferences) and derived the weak-form prediction, a modified version of the (Brunnermeier and Nagel, 2008) theoretical prediction. The weak-form prediction considers two different groups of households. One group of households do not suffer from large negative labor income shocks and the other do. The weak-form prediction suggests that risky shares will increase with a larger magnitude in the group of households without suffering negative labor income shocks than the other group.

We show with the first net effect that for households with the GHH preference, their risky shares decrease with financial wealth, even with a decreasing relative risk aversion or in the absence of negative income shocks. Our finding of the first net effect is in sharp contrast to the theoretical predictions associated with other preferences in the literature, as mentioned in the above and thus is an important contribution to the literature.

[6] Here we assume that $W - C$ is positive. In our empirical exercise, we only use data in which households have positive wealth.

Second, our results shed light on another important and classical question about how labor income theoretically affects optimal portfolio choices, especially with respect to risky shares. The question has been extensively studied [see Bodie et al. (1992), Danthine and Donaldson (2002), Bodie et al. (2004), Munk and Sorensen (2010), among many others]. For example, Bodie et al. (1992) showed the labor income insurance effect, Henderson (2005) studied the optimal portfolio choice problem of an investor with negative exponential utility and facing imperfectly hedgeable stochastic income, and Franke et al. (2011) studied how uncertain labor income affects optimal portfolio choice. The consensus is that the inclusion of labor income has large effects on optimal portfolio choices in theoretical models, and the exact way labor income changes optimal portfolio choices depends on many factors. Our second net effect shows that whether risky shares increase with labor income depends on the key structural parameter, ω. Our result holds even if labor income is constant (i.e., there are no labor income risks at all). Note that both Brunnermeier and Nagel (2008) and Liu et al. (2016) do not discuss this classical question.

2.5. Additional discussions

Our practice of using a parsimonious portfolio choice model to obtain the closed form solution follows Brunnermeier and Nagel (2008) and Liu et al. (2016). The big benefit and advantage of having analytical solutions like (1) are that they allow for robust comparative static analysis in a way that numerical solutions cannot achieve as addressed in Henderson (2005). As we show in the above, our analytical solution in (1) provides ample insights to the literature, while it is hard to show the same insights in such a simple and straightforward manner with numerical solutions.

Clearly, our model is restrictive, especially with the assumptions imposed in Section 2.3. In this regard, more complicated models provide more details. There are ways to complicate the model. The first way can be to consider other preferences, such as recursive preferences [see Bansal et al. (2007)], or non-rational expectation preferences [see Ju and Miao (2012), Guidolin and Liu (2016), Liu (2011)], or kink utilities [see Dahlquist et al. (2017)]. The GHH preference, by construction, neutralizes the wealth effect on labor supply, and labor input is determined independent of the intertemporal consumption and saving choices. Alternative preferences may bring the intertemporal dynamics into labor input determination and thus potentially bring more insight on the two classical portfolio choice problems [see Ai et al. (2018), Croce et al. (2021)]. With the techniques introduced by Campbell and Viceira (2002), approximated solutions can be obtained and the corresponding theoretical discussions can be more comprehensive. The second way can be to move from the simple static analysis with the simple model here to extensive numerical simulation exercise with more complete models. For example, we can replace the assumptions imposed in Section 2.3 with more realistic ones. The third way can be to allow a role for heterogeneity.

Nevertheless, in this paper, our approach is to present our discussion in a way as simple as possible. Thus, we defer studies with more complicated models to our future research.[7]

3. Empirical analysis

In this section, we present data, regression specifications, econometric strategies, and regression analysis results.

3.1. Data

PSID is a national study of socioeconomics over lifetimes and across generations. The study began in 1968 with a nationally representative sample of over 18,000 individuals living in 5000 families in the United States. The data cover many aspects of households, such as employment, income, financial wealth, expenditure, etc. The households' asset holdings are not measured every year. Instead, they are measured in years 1984, 1989, 1994, 1999, 2001, 2003, 2005, 2007, 2009, 2011, 2013, and 2015. Thus, we divide the data into two subsamples: the 1984–1999 ($k = 5$) subsample and the 1999–2015 ($k = 2$) subsample. We apply similar sampling criteria as in Brunnermeier and Nagel (2008) to the PSID data to obtain our samples. Detailed descriptions about sampling criterion are provided below along with the benchmark regression equation provided in Section 3.2.

3.1.1. Key variables

There are three key variables in our empirical models: financial wealth (W), risky shares (α), and labor income (Y). Financial wealth is defined as the sum of liquid wealth, equity in a private business, and home equity. Liquid wealth is herein defined as the difference between liquid assets (which are the sum of risk-free assets and the holdings of stocks and mutual funds) and liquid liabilities, while risk-free assets are defined as the sum of cash-like assets and holdings of bonds. While risky wealth is defined as the summation of liquid wealth (taking risk-free assets away), equity in a private business, and home equity, risky share is defined as the ratio of risky wealth over financial wealth.

Labor income is defined as the labor income of a household. In particular, the value for "labor income" represents the sum of the household head's labor income and spouse's labor income. The value for head's labor income comprises labor part of farm income, labor part of business income, head's wages income, head's bonuses, overtime, commissions, head's income from professional practice or trade, labor part of market gardening income, and labor part of roomers and boarders income. If the spouse had any income from farming, business, market gardening, or roomers and boarders, labor–asset splits were made following the same rules as those for the head. The labor portion of such income is included in the spouse's labor income. We include labor income because wealth in our theoretical model is defined as financial wealth.

[7] We thank a referee to bring the above issues to our attention.

Table 1
Descriptive statistics.

The 1984–1999 Subsample (k = 5)						
Variables	α	$\Delta_k\alpha$	W	$\Delta_k w$	Y	$\Delta_k y$
Statistics						
Mean	0.744	0.047	$676,210	0.315	$98,816	−0.432
Standard Deviation	0.362	0.392	$1,809,472	0.848	$103,917	0.726
Minimum	−2.500	−2.686	$−159,684	−6.815	$217	−6.451
25%	0.581	−0.122	$141,101	−0.110	$51,207	−0.691
50%	0.788	0.030	$288,034	0.323	$78,924	−0.405
75%	0.932	0.196	$626,881	0.742	$112,945	0.001
90%	0.993	0.410	$1,216,057	1.188	$163,918	0.228
Maximum	7.000	6.134	$3.0e+7	4.338	$1,189,108	2.694
N	1,416	1,416	1,417	1,407	1,417	1,412
The 1999–2015 Subsample (k = 2)						
Variables	α	$\Delta_k\alpha$	W	$\Delta_k w$	Y	$\Delta_k y$
Statistics						
Mean	0.740	0.008	$738,389	0.090	$117,189	0.023
Standard Deviation	1.082	1.353	$2,002,355	0.738	$199,364	0.654
Minimum	−49.000	−49.405	$−664,557	−5.941	$1.962	−9.018
25%	0.600	−0.113	$149,680	−0.211	$51,211	−0.129
50%	0.792	0.005	$332,996	0.101	$84,728	0.029
75%	0.936	0.129	$709,021	0.427	$132,082	0.226
90%	0.997	0.308	$1,411,008	0.850	$203,841	0.636
Maximum	20.304	49.696	$4.3e+7	4.684	$8,650,498	6.640
N	6,152	6,152	6,154	6,081	6,154	6,110

3.1.2. Sample selection

Detailed information on our sample selection for year t is given as follows. First, we keep every household that the marital status of the head was the same from year $t-k$ to year t. Second, we keep every household that had not moved from year $t-k$ to year t. Third, we keep every household whose head did not retire in year t. Fourth, we keep every household that participated in the stock market in year t. Fifth, we keep every household whose labor income was positive in year t. Finally, we keep every household that had enough financial wealth ($> \$10,000$) in year $t-k$. Finally, define $\Delta_k\alpha = \alpha_t - \alpha_{t-k}$ to denote the change of risky shares, the difference between the current year, and the k years lagged behind the current year. Similarly, we define $\Delta_k w = \log(W_t) - \log(W_{t-k})$ to be the change of logarithms of financial wealth and $\Delta_k y = \log(Y_t) - \log(Y_{t-k})$ to be the change of logarithms of labor income.

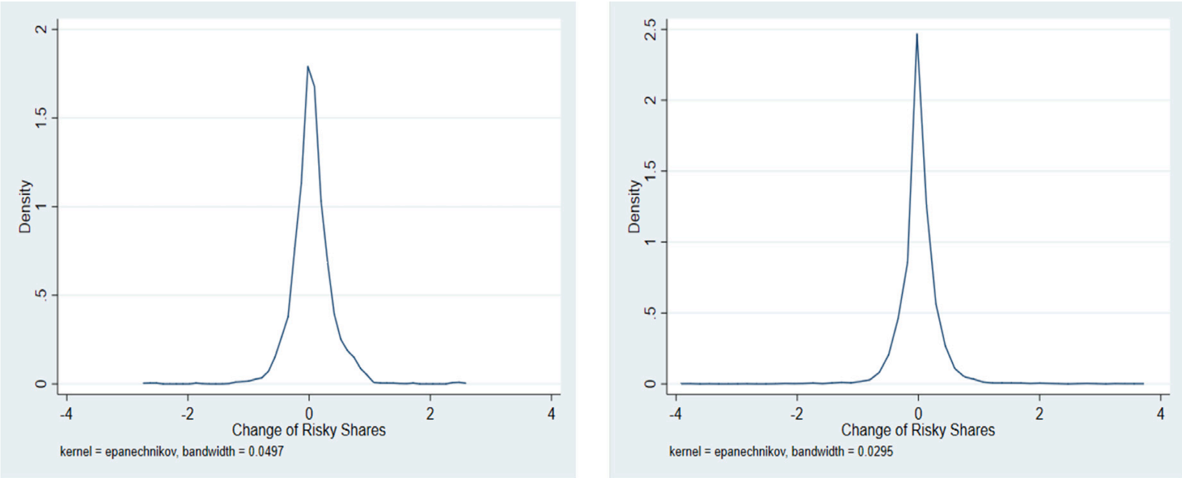
3.1.3. Descriptive analyses

Table 1 presents some descriptive statistics of risky shares,

financial wealth, and labor income of households. The column under α shows the descriptive statistics of risky shares in both subsamples. From the table, the average of risky shares in the 1984–1999 subsample is around 74.4%, which is slightly higher than that (74.0%) in the 1999–2015 subsample. The median of risky shares is about 78.8% in the 1984–1999 subsample, which is slightly lower than that (79.2%) in the 1999–2015 subsample. These statistics suggest that, on average, households invest about 3 quarters of their financial wealth in risky assets (such as stocks) over time. The statistics are close to these in Brunnermeier and Nagel (2008) while are higher than the typical used values, approximately 50% among stock participants, in the Survey of Consumer Finances data [see Vissing-Jorgensen (2002)]. The reason is because we follow the definition of financial wealth in Brunnermeier and Nagel (2008) and include home equity when we calculate financial wealth; while those studies with lower numbers do not include home equity.

Also, the column under W shows the descriptive statistics of financial wealth. The average of financial wealth in the 1984–1999 subsample is $676,210, which is smaller than that ($738,389) in the 1999–2015 subsample. The median of financial wealth is $288,034 in the 1984–1999 subsample, which is also smaller than that ($332,996) in the 1999–2015 subsample. The column under Y shows the descriptive statistics of labor income. The average of labor income in the 1984–1999 subsample is $98,816, which is smaller than that ($117,189) in the 1999–2015 subsample. The median of labor income is $78,924 in the 1984–1999 subsample, which is also smaller than that ($84,728) in the 1999–2015 subsample. Generally, households become richer and earn more labor income over time. Most households are concentrated on the left ends of the distributions of both financial wealth and labor income over time as well.

Furthermore, all variables seem to have influential observations (outliers) when one compares the minimums and maximums to the mean or the median. For example, the distributions of risky shares are over an extremely large spectrum. The minimum risky share is as low as −2.5 (i.e., −250%) and as high as 700% in the 1984–1999 subsample. The distribution becomes even more volatile in the 1999–2015 subsample, i.e., the minimum is as low as −49.0 (i.e., −4900%) and the maximum is as high as 20.3 (i.e., 2030%). Those extreme values exist for various reasons. One possibility is that a household's home equity may turn negative. For example, the value of one household's house was below -$110,000, while its financial wealth was barely over $2000. As a result, the household's risky share was −4900%. Another scenario could be that households borrowed too much so their financial



(a) (b)

Fig. 1. Density of $\Delta_k \alpha_t$ in two subsamples with Panel (a) for the 1984–1999 subsample and Panel (b) for the 1999–2015 subsample.

Table 2
Skewness/Kurtosis tests for normality.

Variable: $\Delta_k \alpha$	The 1984–1999 Subsample (k = 5)	The 1999–2015 Subsample (k = 2)
Prob(Skewness)	0.0000	0.0000
Prob(Kurtosis)	0.0000	0.0000
$\chi^2(2)$	1619	3618
Prob > χ^2	0.0000	0.0000
Sample size	1,416	6,152

wealth became negative. For example, a household had about $9000 risky assets, while its financial wealth was -$830. Together, this household's risky share was about −1100%. For another example, a household's financial wealth was about $19,200 while its risky assets were about $390,000. Thus, the household's risky share was about 2030%. Clearly, all examples mentioned here are extreme cases and all of them lie outside three standard errors from the mean. Other two variables, financial wealth and labor income, contain outliers as well.

Again, from Table 1, there are interesting observations of these statistics for the key variables of interest $\Delta_k \alpha_t$, $\Delta_k w$, and $\Delta_k y_t$. For example, the top 10% households increased their risky shares by 41.0 percentage points from year $t-k$ to year t in the 1984–1999 subsample and by 30.8 percentage points in the 1999–2015 subsample. Both increases are far larger than the corresponding changes made by the mode household. However, the bottom 25% households decreased their risky shares. Similar patterns as that of $\Delta_k \alpha_t$ emerge in the changes of financial wealth, $\Delta_k w$, and the changes of labor income, $\Delta_k y_t$. One may view these observations as a piece of evidence showing that the gap among households had been getting wider and wider from 1984 to 2015.

Fig. 1 shows the density distributions of $\Delta_k \alpha_t$ in the two subsamples. Panel (a) shows the density distribution of $\Delta_k \alpha_t$ in the range of −4 and 4 in the 1984–1999 subsample and Panel (b) presents the density distribution of $\Delta_k \alpha_t$ in the range of −4 and 4 in the 1999–2015 subsample. As it is clear from the density distributions that $\Delta_k \alpha_t$ is not normally distributed in both subsamples, we further perform the skewness and kurtosis normality tests of $\Delta_k \alpha_t$ in both subsamples. The test results are reported in Table 2.

The p-values of the skewness tests are close to zero in both subsamples. This result rejects the symmetry hypothesis of the distribution of $\Delta_k \alpha_t$ in both subsamples. The p-values of the kurtosis tests are also close to zero in both subsamples. This result means that the distributions of $\Delta_k \alpha_t$ in both subsamples have fatter tails than normal distributions. Given the skewness and kurtosis test results, quantile regressions are necessary to handle the outlier and fat tail issues in the distributions of $\Delta_k \alpha_t$.

Fig. 2 shows scatter plots of $\Delta_k \alpha_t$ against $\Delta_k w$ and $\Delta_k y_t$ in the two subsamples. In all panels, the vertical axis represents the absolute value of the change of risky shares. The horizontal axes in Panels (a) and (c) represent the change of financial wealth and the horizontal axes in Panels (b) and (d) give the change of labor income. Panel (a) is the scatter plot of $\Delta_k \alpha_t$ and $\Delta_k w$ in the 1984–1999 subsample and Panel (b) is the scatter plot of $\Delta_k \alpha_t$ and $\Delta_k y_t$ in the 1984–1999 subsample. Panel (c) is the scatter plot of $\Delta_k \alpha_t$ and $\Delta_k w$ in the 1999–2015 subsample and Panel (d) is the scatter plot of $\Delta_k \alpha_t$ and $\Delta_k y_t$ in the 1999–2015 subsample. The extreme distributions of $\Delta_k \alpha_t$, $\Delta_k w$ and $\Delta_k y_t$ in the two subsamples are presented in these four panels. First, given the presence of outliers of risky shares, financial wealth, and labor income, it is not surprising that the changes in these three variables also contain

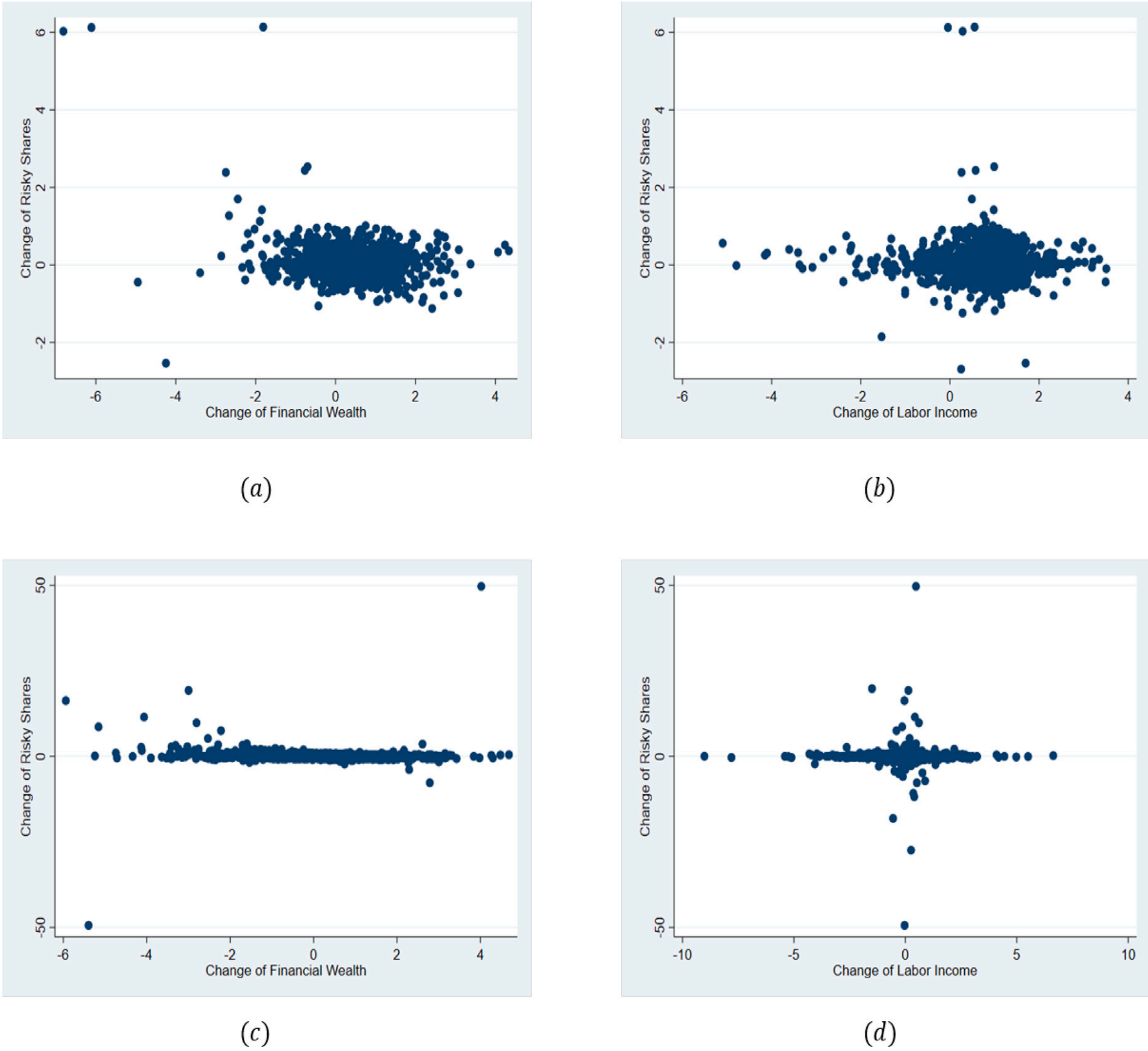


Fig. 2. Scatter plots for the absolute value of $\Delta_k \alpha_t$ against $\Delta_k w$ in the left panel and $\Delta_k y$ in the right panel, with the top panel for the 1984–1999 subsample and the bottom panel for the 1999–2015 subsample.

outliers. Second, the scatter plots do not present a clear relationship between $\Delta_k \alpha_t$ and $\Delta_k w$ (or $\Delta_k y_t$). However, once we draw scatter plots without the naive outliers (observations that lie outside three standard errors from the mean), a weak negative relationship between $\Delta_k \alpha_t$ and $\Delta_k w$ is presented.

Given the ambiguous patterns suggested by the scatter plots, it is necessary to run regressions to figure out the relationships among these variables in the data. The descriptive statistics indicated that the empirical data contain significant outliers. Also based on the density distributions, $\Delta_k \alpha_t$ have fat tails. We use two strategies to address the outlier and fat tail issues. The first approach is to run regressions with and without outliers. We adopt a straightforward definition of outliers: observations that lie outside three standard errors from the mean. This is due to the fact that the proportion of observations that lie outside three standard errors from the mean is less than 2.0% with respect to $\Delta_k \alpha_t$, $\Delta_k w$ and $\Delta_k y_t$.[8] The second approach is to run quantile regressions over two full subsamples.

[8] We also use an alternative definition of outliers of $\Delta_k \alpha$. According to the settings of our theory model, we expect that the minimum of $\Delta_k \alpha$ be −100% and the maximum be 100%. Further check shows that $\Delta_k \alpha_t$ below −100% or above 100% is less than 1.4% in the 1984–1999 subsample and less than 2.0% in the 1999–2015 subsample. Given the low proportion, if we treat observations whose $\Delta_k \alpha$ are below −100% or above 100% as outliers, our main regression results remain unchanged.

3.2. Regression specification and strategy

We consider the following regression equation:

$$\Delta_k \alpha_t = \rho \, \Delta_k w_t + \gamma \, \Delta_k y_t + \beta \, q_{t-k} + \zeta \, \Delta_k n_t + \varepsilon_t, \tag{6}$$

where α_t denotes the risky share and is defined as the proportion of financial wealth invested in stocks, mutual funds, equity in a private business, and home equity in year t, $\Delta_k \alpha_t$ denotes the change of risky shares of the household j over the k years, $\Delta_k w_t$ denotes the change of log of financial wealth over the k years, and $\Delta_k y_t$ represents the change of log of labor income over the k years. Here, q_{t-k} is a vector of household characteristics and the fixed time effects for the household. For example, it includes a broad range of variables related to the life cycle, background, and financial situation of the household. The vector $\Delta_k n_t$ contains variables that capture major changes in household characteristic or asset ownership. For example, it includes changes in family size, changes in the number of children, and dummies for house ownership, business ownership, and non-zero labor income at t and $t-k$. We estimate the model in (6) and its various versions, in both the 1984–1999 ($k = 5$) subsample and the 1999–2015 ($k = 2$) subsample, to obtain the estimates of ρ and γ.

As emphasized in Betermier et al. (2012), a major challenge for empirical analyses on this topic is that events may exist and cause both changes in income/wealth and risky shares. For example, it is possible that both α_t and w_t/y_t have some correlated deterministic pattern over the life cycle (Brunnermeier and Nagel, 2008). We take three steps to account for this challenge. In the first step, we impose the following sample criterion (as shown in Section 3.1.2): the marital status of the family unit head remained unchanged, the household head did not retire, and the household did not move, from year $t-k$ to year t. In the second step, we introduce control variables such as q_{t-k} [as explained in the above and right below (6)] and $\Delta_k n_t$ [also as explained in the above and right below (6)] in the regression model. The inclusion of these additional variables serves the purpose of controlling for some important variables, such as life-cycle effects and preference shifters, and idiosyncratic versus aggregate financial wealth changes, that may cause changes to both α_t and w_t/y_t. In the last step, we use the difference method, and for example, we use $\Delta_k \alpha_t$ instead of α_t.

The second challenge is that the data may contain measurement errors. We use a two-stage least square estimator to account for potential measurement errors. The identification requirement is that the instruments, IVs, are (partially) correlated with $\Delta_k w_t$ (and/or $\Delta_k y_t$), but not correlated with the error terms. In this paper, we adopt three instrumental variables for changes in financial wealth and labor-income growth, including one from Liu et al. (2016) and two other instruments from Brunnermeier and Nagel (2008). We explain these instruments right below. In Section 3.3, we show that the instruments satisfy the identification requirements, (partially) correlated with $\Delta_k w_t$ (and/or $\Delta_k y_t$), but not correlated with the error terms.

The instrumental variable from Liu et al. (2016) can be regarded as the growth rate of the ratio of the household head's labor income to a measure of wealth. Mathematically, the instrument variable is given by $dlabfw = \log(labfw/llabfw)$, where $labfw = hdlabinc5/(fw + svodbt)$ and $llabfw = lhdlabinc5/(lfw + lsvodbt)$. Here, $hdlabinc5$ denotes household head's labor income in the current year, fw denotes liquid wealth in the current year, $svodbt$ denotes the dollar value of other debts in the current year (other debt comprises of non-mortgage debt such as credit card debt and consumer loans), and $(fw + svodbt)$ denotes liquid assets in the current year. And $lhdlabinc5$, lfw, $lsvodbt$, and $(lfw + lsvodbt)$ are the lagged $hdlabinc5$, fw, $svodbt$, and $(fw + svodbt)$ by k years. Since the instrument is obtained using data about income and wealth, it is reasonable to assume that this instrument is partially correlated with financial wealth fluctuations and labor income changes. The performance of the instrument variable is discussed below in Section 3.3.

The two instrument variables from Brunnermeier and Nagel (2008) are dummy variables for income growth that is measured independently of financial wealth, $dincd1$ and $dincd2$.[9] In particular, $dincd1 = 1$ if the household's income growth (k-years ago) is in the lowest decile, 0 otherwise, and $dincd2 = 1$ if the household's income growth (k-years ago) is in the top decile, 0 otherwise. As emphasized in Brunnermeier and Nagel (2008), the values of these two instrument variables are based on survey questions that are different from those for w_t, and it is thus reasonable to assume that they are uncorrelated with the error terms. Again, the performance of these instrument variables are discussed below in Section 3.3.

3.3. Benchmark regression results

In this section, we report the regression results (as shown in Tables 3 and 4) in the following order. First, we present the performance assessment of the instrumental variables. Second, we discuss the results of how financial wealth affects risky shares. Third, we present the results of how labor income affects risky shares. Finally, we report the results of the impact of outliers and fat tails.

[9] The income here is not the labor income.

Table 3
Benchmark regression results.

(A) First Stage Results	The 1984–1999 Subsample (k = 5)	The 1999–2015 Subsample (k = 2)
Dependent variable: $\Delta_k w_t$		
Key explanatory variables of interest:		
IV: dlabfw	−0.348***	−0.373***
	(0.022)	(0.012)
IV: dincd1	0.485***	−0.421***
	(0.093)	(0.032)
IV: dincd2	0.388***	0.346***
	(0.069)	(0.034)
R^2	0.410	0.379
Sample size	1,407	6,081
(B) First Stage Results	**The 1984–1999 Subsample (k = 5)**	**The 1999–2015 Subsample (k = 2)**
Dependent variable: $\Delta_k y_t$		
Key explanatory variables of interest:		
IV: dlabfw	0.116***	0.124***
	(0.015)	(0.010)
IV: dincd1	−0.751***	−0.731***
	(0.083)	(0.032)
IV: dincd2	0.398***	0.655***
	(0.065)	(0.029)
R^2	0.427	0.419
Sample size	1,412	6,110
(C) Second Stage Results	**The 1984–1999 Subsample (k = 5)**	**The 1999–2015 Subsample (k = 2)**
Dependent variable: $\Delta_k \alpha_t$		
Key explanatory variables of interest:		
$\Delta_k w_t$ ($\hat{\rho}$):	−0.178***	−0.163***
	(0.025)	(0.048)
$\Delta_k y_t$ ($\hat{\gamma}$):	−0.020	0.051
	(0.036)	(0.045)
Sample size	1,402	6,038
Weak instrument test		
Cragg–Donald Wald F statistic	95	656
Kleibergen–Paap rk Wald F statistic	53	295
10% significance critical value	13.43	13.43

Notes: The benchmark regression equation and all IVs are defined in Section 3.2. Heteroskedasticity and autocorrelation-robust standard errors are used to judge the significance of estimates. Their values are shown in parenthesis. *** (**, *) means that the estimate is statistically significantly different from 0 at the 1% (5%, 10%) significance level, respectively.

3.3.1. *Performance of the instruments*

According to the estimation results, it is evident that our instrumental variables are correlated to the financial wealth fluctuations and labor income changes. The results in Panel (A) of Table 3 show that the instruments explain a reasonable fraction of variation in financial wealth fluctuations. The instruments are statistically significant, with p-values smaller than 1%. The partial R^2 of the instruments is about 0.410 for the 1984–1999 ($k = 5$) subsample and is about 0.379 for the 1999–2015 ($k = 2$) subsample.

Based on the estimation results, our instrumental variables are also correlated to the labor income changes. The results in Panel (B) of Table 3 show that the instruments have a statistically significant partial correlation with changes in log labor income. The instruments are statistically significant, with p-values smaller than 1%. The partial R^2 of the instruments is about 0.427 for the 1984–1999 ($k = 5$) subsample and is about 0.419 for the 1999–2015 ($k = 2$) subsample.

The results in Panel (C) of Table 3 demonstrate that our instruments pass weak-instrument tests. In particular, the value of the Cragg–Donald Wald F statistic is 95 in the 1984–1999 ($k = 5$) subsample and is 656 in the 1999–2015 ($k = 2$) subsample, respectively. Since we estimate by clustering data with family IDs, the robust statistic is the Kleibergen–Paap Wald F statistic [see Kleibergen and Paap (2006)].[10] The value of the Kleibergen–Paap Wald F statistic is 53 in the 1984–1999 ($k = 5$) subsample and is 295 in the 1999–2015 ($k = 2$) subsample, respectively. All of them are way larger than the 10% Stock–Yogo weak ID test critical value, 13.43. Thus, we reject the hypothesis that our instruments are weak.

3.3.2. *Responses to financial wealth accumulations*

How risky shares respond to financial wealth accumulations is a classical question. According to classical economic theory, if a household has constant relative risk aversion (hereafter CRRA) preferences, risky shares are a constant and thus do not respond

[10] According the STATA manual: "When the i.i.d. assumption is dropped and ivreg2 is invoked with the robust, bw or cluster options, the Cragg–Donald-based weak instruments test is no longer valid. ivreg2 instead reports a correspondingly-robust Kleibergen–Paap Wald F statistic".

Table 4
Impact of outliers and asymmetry/Fat tails.

(A) (6) without outliers		The 1984–1999 Subsample ($k = 5$)	The 1999–2015 Subsample ($k = 2$)
Dependent variable: $\Delta_k \alpha_t$			
Key explanatory variables of interest:			
$\Delta_k w_t$ ($\hat{\rho}$):		−0.175***	−0.157***
		(0.028)	(0.012)
$\Delta_k y_t$ ($\hat{\gamma}$):		−0.050	−0.001
		(0.040)	(0.015)
Sample size		1,343	5,795
Weak instrument test			
Cragg–Donald Wald F statistic		102	773
Kleibergen–Paap rk Wald F statistic		61	393
10% significance critical value		13.43	13.43
(B) (7) for quantile regressions		**The 1984–1999 Subsample ($k = 5$)**	**The 1999–2015 Subsample ($k = 2$)**
Dependent variable: $\Delta_k \alpha_t$			
Key explanatory variables of interest:			
$\Delta_k w_t$ ($\hat{\rho}$):	(10%)	−0.132***	−0.106***
	(25%)	−0.142***	−0.139***
	(50%)	−0.179***	−0.138***
	(75%)	−0.189***	−0.150***
	(90%)	−0.211***	−0.176***
$\Delta_k y_t$ ($\hat{\gamma}$):	(10%)	−0.016	−0.025***
	(25%)	−0.011	0.004
	(50%)	−0.024	−0.005
	(75%)	−0.043	0.003
	(90%)	−0.022	0.023***
Sample size		1,402	6,038

Notes: All the results for the second-stage regression results are presented in Panel (A) are and all the results for the quantile regression results are summarized in Panel (B). Heteroskedasticity and autocorrelation-robust standard errors are used to judge the significance of estimates. *** (**, *) means that the estimate is statistically significantly different from 0 at the 1% (5%, 10%) significance level, respectively.

to financial wealth accumulations [see Samuelson (1969)]. Sahm (2012) found an evidence of CRRA preferences in the 1992–2002 Health and Retirement Study data. Brunnermeier and Nagel (2008) showed that if a household has habit-formation preferences, its risky shares should increase when its wealth accumulates. However, Brunnermeier and Nagel (2008) found a negative response of risky shares to financial wealth accumulation in the 1984–2003 PSID data, which deviates from the prediction of habit-formation preferences. Liu et al. (2016) proved that a household with habit-formation preferences may increase its risky shares in responding to financial wealth accumulation at a slower rate if it suffers large negative income shocks than otherwise. Liu et al. (2016) found predicted heterogeneous responses in the 1984–1999 PSID data, which is in line with their heterogeneous prediction.

With the instruments, the results in Panel (C) of Table 3 find significant negative responses of risky shares to financial wealth accumulations in both the 1984–1999 ($k = 5$) subsample and the 1999–2015 ($k = 2$) subsample. For example, the estimates of the responses, $\hat{\rho}$, are −0.178 and −0.163 in the 1984–1999 ($k = 5$) subsample and the 1999–2015 ($k = 2$) subsample, respectively. Both are statistically significantly different from zero at the 1% significance level. These estimates are economically significant as well and they are close in terms of magnitude to the estimates in related studies, such as Brunnermeier and Nagel (2008) and Liu et al. (2016).

With respect to this classical question, we contribute to the literature by providing several important new findings. First, we provide new theoretical evidence, as shown in Section 2.4.2, to imply that under a standard calibration of the key parameter ω, a household with the GHH preference reduces (as the net effect) its risk tolerance (i.e., decrease its risky shares) when it accumulates financial wealth. This theoretical finding is in sharp contrast to the aforementioned studies. Second, we obtain new empirical results regarding this classical problem. Specifically, although our 2SLS estimates are generally negative, they are quite robust based on the 1984–2015 PSID data, and they are both statistically and economically significant. In contrast, Liu et al. (2016) only used the 1984–1999 PSID data, while (Brunnermeier and Nagel, 2008) only used 1984–2003 PSID data. Third, more importantly, our comprehensive and significant negative estimates are in line with the hypothesis that $\rho < 0$ as predicted with our theoretical model. In other words, our empirical results regarding how risky shares respond to financial wealth fluctuations provide very strong empirical support of the GHH preference. Overall, we obtain new empirical results and provide the underlying theory that is compatible with the empirical results with respect to this classical problem.

3.3.3. Responses to labor income changes

How risky shares respond to labor income changes remains another classical question. According to the theoretical discussions in Bodie et al. (1992), labor income provides an insurance to the households against adverse investment outcome, and households thus increase their risky shares with higher labor income. Empirical studies with aggregate data (with noisy measurements) find mixed evidence [see Fama and Schwert (1977), Lustig and Nieuwerburgh (2008), among many others]. Existing empirical studies with household-level data have found flat responses of risky shares to labor income [see Guiso et al. (1996), Heaton and Lucas (2000), Betermier et al. (2012), among others].

With the instruments, the results in Panel (C) of Table 3 show (insignificant) responses of risky shares to labor income changes in both the 1984–1999 ($k = 5$) subsample and the 1999–2015 ($k = 2$) subsample. For example, the estimates of the responses, $\hat{\gamma}$, are −0.020 in the 1984–1999 ($k = 2$) subsample and 0.051 in the 1999–2015 ($k = 2$) subsample, respectively. The estimates are statistically insignificant. The results from the 1999–2015 sample provide very weak empirical evidence, if there is any, in support of the GHH preference.

With respect to this classical question, we have also made important progress. First, we bring new insight to the literature. The main theoretical contribution, as we have shown in Section 2.4.2, is that under the standard calibration of the key parameter ω, a household with the GHH preference increases (as the net effect) its risk tolerance (i.e., increase its risky shares) when its labor income increases. Second, we document new empirical results regarding how labor income affects risky shares. Based on our 2SLS estimate using the 1984–2015 PSID data, we find that risky shares do not respond to the labor income growth. Even though our estimates regarding how labor income affects risky shares are in line with existing literature [see Guiso et al. (1996), Heaton and Lucas (2000), Betermier et al. (2012), among others], they are different from our theoretical predictions. We take this separation of the empirical results and the theoretical implications as the indicator that the modeling of labor income should be improved.

3.4. Quantile analysis

From the data presented in Section 3.1.3, there might exist some extreme cases of risky shares variations, financial wealth fluctuations, and labor income growth changes, so that the distributions of $\Delta_k \alpha_t$ seem to have fat tails in both subsamples. The benchmark results may be distorted by those extreme values and fat tails. In this section, we use two different approaches to analyze the impact of the extreme values and fat tails on our results, and to account for the effects due to the distribution of endogenous variables. Firstly, we re-estimate the mean model in (6) after removing outliers, which are defined with a simple threshold: any observation that lies outside three standard errors from the mean. With this naive definition of outliers, we remove outliers among $\Delta_k \alpha_t$, $\Delta_k w_t$ and $\Delta_k y_t$. With the second approach, we run quantile regressions by estimating the following equation:

$$q_\tau(Q_t) = \rho_\tau \, \Delta_k w_t + \gamma_\tau \Delta_k y_t + \beta_\tau \, q_{t-k} + \zeta_\tau \, \Delta_k n_t, \tag{7}$$

where Q_t contains $\Delta_k w_t$, $\Delta_k y_t$, q_{t-k} and $\Delta_k n_t$; and $\Delta_k w_t$ and $\Delta_k y_t$ are endogenous variable. Quantile regressions are a natural way to test the impact of outliers, or more generally, the cases at the highest or lowest quantile (i.e., fat tails), on the relationships of interest. Furthermore, quantile regressions directly show the effects of the distributions of wealth and labor income. To estimate the parameters in (7), i.e., running quantile regression with the instrumental variables, we use the **genqreg** package of Stata, which implements the generalized quantile estimator based on the work of Powell (2020).[11] In particular, for each quantile, we run the **genqreg** regression with the same instrumental variables. For more discussions about quantile regressions with instrumental variables, please see Chernozhukov and Hansen (2008), Powell (2020), and many others.

Panel (A) of Table 4 shows the second-stage regression results of running the 2SLS regression of (6) without outliers. The responses of risky shares to financial wealth fluctuations are pretty much the same as in the case with outliers included. For example, the estimates of the responses, $\hat{\rho}$, are −0.175 and −0.157 in the 1984–1999 ($k = 5$) subsample and the 1999–2015 ($k = 2$) subsample, respectively. These estimates are statistically significant at the 1% level and their magnitudes are very close to −0.178 and −0.163 (the estimates associated with the case that includes outliers), respectively. Thus, outliers do not affect the relationship between risky shares and financial wealth fluctuations in the data. The responses of risky shares to labor income changes are pretty much the same as in the case with outliers included. For example, the estimates of the responses, $\hat{\gamma}$, are −0.050 and −0.001 in the 1984–1999 ($k = 5$) subsample and the 1999–2015 ($k = 2$) subsample, respectively. These estimates are statistically insignificant as in the case with outliers included. Outliers also have no effect on the relationship between risky shares and labor income in the data. Overall, outliers in the empirical data do not have a substantive effect on the estimated household portfolio choices.

Panel (B) of Table 4 shows the quantile regression results of (7) without removing outliers. The responses of risky shares to financial wealth fluctuations across all quantiles are pretty much the same as the corresponding second-stage 2SLS regression results: they are statistically significant at the 1% level and their magnitudes are very close to −0.178 and −0.163 (the estimates at median), respectively. In the 1984–1999 ($k = 5$) subsample, the estimate of the responses, $\hat{\rho}$, is −0.132 at the lowest 10th percentile, becomes more negative as the quantile climbs up and reaches the highest, −0.211 at the 90th percentile. The quantile regression results about $\hat{\rho}$ in the 1999–2015 ($k = 2$) subsample are very close to those in the 1984–1999 ($k = 5$) subsample. The estimate of the responses, $\hat{\rho}$, is −0.106 at the lowest 10th percentile, becomes more negative as the quantile climbs up and reaches the highest, −0.176 at the 90th percentile. Thus, cases at the highest or lowest quantile (including outliers about portfolio choice adjustments) do not affect the relationship between risky shares and financial wealth fluctuations in the data.

The responses of risky shares to labor income changes are also pretty much the same as the corresponding second-stage 2SLS regression results: statistically insignificant and quantitatively close to zero. For the 1984–1999 ($k = 5$) subsample, the estimate of the responses, $\hat{\gamma}$, is −0.016 at the 10th percentile, and it is statistically insignificant. Across all quantiles, the estimates, $\hat{\gamma}$, are

[11] For more discussions about this package, please check the following website: https://www.statalist.org/forums/forum/general-stata-discussion/general/1331987-new-packages-on-ssc-genqreg-and-qregpd-generalized-quantile-regression-and-quantile-regression-with-panel-data.

statistically insignificant. For the 1999–2015 ($k = 2$) subsample, the estimates are slightly different in the sense that estimates at the highest or lowest quantile are statistically significant. In this subsample, the estimate at the 10th percentile is −0.025 and statistically significant at the 1% level, and the estimate at the 90th percentile is 0.023 and also statistically significant at the 1% level. Overall, the estimates are statistically insignificant as in the case that includes outliers. Thus, cases at both the highest and lowest quantiles (including outliers about portfolio choice adjustments) do have an impact on the relationship between risky shares and labor income in the data. In other words, the conditional Value-at-Risk (VaR) of risky shares depends on both financial wealth and labor income. Therefore, the existence of fat tails in the distributions of $\Delta_k \alpha_t$ helps explain the significant results in both the highest and lowest quantiles. This finding seems to be new in the literature.

4. Conclusion

In this paper, we employ a parsimonious household portfolio choice model with the GHH preference to explore the underlying mechanisms of portfolio choices, using the 1984–2015 PSID data. We contribute to the literature, particularly macroeconomics and finance literature, via four dimensions. First, we obtain a closed-form solution to risky shares. Second, we derive clear theoretical predictions of portfolio choices when household preferences are the GHH preference. Third, we check the theoretical implications with empirical data. Our empirical results strongly support the predictions of how financial wealth accumulations affect portfolio choices while they do not support the predictions regarding how labor income affects portfolio choices. Finally, our results show that the mechanism generated by our model with the GHH preference is a plausible approach to understanding portfolio choices in the PSID data. This approach, nevertheless, needs to be modified in order to further capture the impact of labor income on portfolio choices.

Our theoretical analysis is intentionally kept simple thus ignoring many important theoretical issues. The paper assumes the GHH preference, a specific preference, thus ignores alternative preferences. The paper imposes restrictive assumptions and they are clearly not realistic. The theoretical analysis of the paper focuses on the static relationships among the three key variables, thus ignoring the impact of the short-run dynamics among these variables. The model in the paper clearly misses the impact of labor income on risky shares. The empirical analysis of the paper focuses on the linear relationships among the three key variables, thus ignoring the non-linear relationships.

Based on our work in this paper, there are several extensions worth exploring in the future. There are many theoretic extensions. The first extension is to consider other preferences such as recursive preferences or non-rational expectation preferences. The second extension is to relax the restrictive assumptions and the model may be more realistic. The third extension is to discuss the short-run dynamics among the variables of interest. The fourth extension is to modify the model in order to account for the impact of labor income on portfolio choices. The fifth extension is to investigate how labor income risks affect portfolio choices. The sixth extension is to have joint discussions on both portfolio choices and consumption. The seventh extension is to study the role of heterogeneity in understanding portfolio choices with the household level data. The eighth extension is to discuss nonlinear relationships among the variables of interest.

CRediT authorship contribution statement

Xuan Liu: Writing – review & editing, Writing – original draft, Formal analysis, Data curation, Conceptualization. **Haiyong Liu:** Writing – review & editing, Supervision, Investigation, Formal analysis, Conceptualization. **Zongwu Cai:** Writing – review & editing, Methodology, Investigation, Formal analysis, Conceptualization.

References

Ai, H., Croce, M.M., Diercks, A.M., Li, K., 2018. News shocks and the production-based term structure of equity returns. Rev. Financial Stud. 31 (7), 2423–2467.
Bansal, R., Gallant, A.R., Tauchen, G., 2007. Rational pessimism, rational exuberance, and asset pricing models. Rev. Econ. Stud. 74, 1005–1033.
Barro, R.J., King, R.G., 1984. Time-separable preferences and intertemporal-substitution models of business cycles. Q. J. Econ. 99 (4), 817–839.
Betermier, S., Jansson, T., Parlour, C., Walden, J., 2012. Hedging labor income risk. J. Financial Econ. 105 (3), 622–639.
Bodie, Z., Detemple, J.B., Otruba, S., Walter, S., 2004. Optimal consumption-portfolio choices and retirement planning. J. Econ. Dyn. Control 28 (6), 1115–1148.
Bodie, Z., Merton, R., Samuelson, W.F., 1992. Labor supply flexibility and portfolio choice in a life cycle model. J. Econ. Dyn. Control 16 (3–4), 427–449.
Brunnermeier, M.K., Nagel, S., 2008. Do wealth fluctuations generate time-varying risk aversion? Micro-evidence from individuals' asset allocation. Am. Econ. Rev. 98 (3), 713–736.
Campbell, J.Y., Viceira, L.M., 2002. Strategic asset allocation: Portfolio choice for long-term investors. In: OUP Catalogue, (9780198296942), Oxford University Press.
Chang, Y., Kim, S.B., 2007. Heterogeneity and aggregation: Implications for labor-market fluctuations. Am. Econ. Rev. 97 (5), 1939–1956.
Chernozhukov, V., Hansen, C., 2008. Instrumental variable quantile regression: A robust inference approach. J. Econom. 142 (1), 379–398.
Chetty, R., 2006. A new method of estimating risk aversion. Am. Econ. Rev. 96 (5), 1821–1834.
Constantinides, G.M., 1990. Habit formation: A resolution of the equity premium puzzle. J. Political Econ. 98 (3), 519–543.
Croce, M.M., Nguyen, T.T., Raymond, S., 2021. Persistent government debt and aggregate risk distribution. J. Financial Econ. 140 (2), 347–367.
Dahlquist, M., Farago, A., Tedongap, R., 2017. Asymmetries and portfolio choice. Rev. Financial Stud. 30 (2), 667–702.
Danthine, J.P., Donaldson, J.B., 2002. Labour relations and asset returns. Rev. Econ. Stud. 69 (1), 41–64.
Fama, E.F., Schwert, G.W., 1977. Human capital and capital market equilibrium. J. Financial Econ. 4 (1), 95–125.
Ferriere, A., Navarro, G., 2018. The Heterogeneous Effects of Government Spending : Its All About Taxes. Technical Report.
Franke, G., Schlesinger, H., Stapleton, R.C., 2011. Risk-taking with additive and multiplicative background risks. J. Econ. Theory 146 (4), 1547–1568.
Garcia-Cicco, J., Pancrazi, R., Uribe, M., 2010. Real business cycles in emerging countries? Am. Econ. Rev. 100 (5), 2510–2531.
Greenwood, J., Hercowitz, Z., Huffman, G.W., 1988. Investment, capacity utilization, and the real business cycles. Am. Econ. Rev. 78, 402–417.

Guidolin, M., Liu, H., 2016. Ambiguity aversion and underdiversification. J. Financial Quant. Anal. 51 (4), 1297–1323.

Guiso, L., Jappelli, T., Terlizzese, D., 1996. Income risk, borrowing constraints, and portfolio choice. Am. Econ. Rev. 86 (1), 158–172.

Heaton, J., Lucas, D., 2000. Portfolio choice and asset prices: The importance of entrepreneurial risk. J. Finance 55 (3), 1163–1198.

Henderson, V., 2005. Explicit solutions to an optimal portfolio choice problem with stochastic income. J. Econ. Dyn. Control 29 (7), 1237–1266.

Jahan-Parvar, M.R., Liu, X., Rothman, P., 2013. Equity returns and business cycles in small open economies. J. Money Credit Bank 45 (6), 1117–1146.

Jermann, U.J., 1998. Asset pricing in production economies. J. Monetary Econ. 41, 257–275.

Ju, N., Miao, J., 2012. Ambiguity, learning, and asset returns. Econometrica 80 (2), 559–591.

Kleibergen, F., Paap, R., 2006. Generalized reduced rank tests using the singular value decomposition. J. Econom. 133 (1), 97–126.

Liu, H., 2011. Dynamic portfolio choice under ambiguity and regime switching mean returns. J. Econ. Dyn. Control 35 (4), 623–640.

Liu, X., Yang, F., Cai, Z., 2016. Does relative risk aversion vary with wealth? Evidence from households' portfolio choice data. J. Econ. Dyn. Control 69, 229–248.

Lustig, H., Nieuwerburgh, S.V., 2008. The returns on human capital: Good news on wall street is bad news on main street. Rev. Financial Stud. 21 (5), 2097–2137.

Mendoza, E.G., 1991. Real business cycles in a small open economy. Am. Econ. Rev. 81 (4), 797–818.

Munk, C., Sorensen, C., 2010. Dynamic asset allocation with stochastic income and interest rates. J. Financial Econ. 96 (3), 433–462.

Neumeyer, P.A., Perri, F., 2005. Business cycles in emerging economies: The role of interest rates. J. Monetary Econ. 52, 345–380.

Powell, D., 2020. Quantile treatment effects in the presence of covariates. Rev. Econ. Statist. 105 (5), 1–39.

Sahm, C.R., 2012. How much does risk tolerance change? Q. J. Finance 2 (04), 1–38.

Samuelson, P.A., 1969. Lifetime portfolio selection by dynamic stochastic programming. Rev. Econ. Statist. 51 (3), 239–246.

Schmitt-Grohé, S., Uribe, M., 2003. Closing small open economy models. J. Int. Econ. 61, 163–185.

Stiglitz, J.E., 1969. Behavior towards risk with many commodities. Econometrica 37 (4), 660–667.

Vissing-Jorgensen, A., 2002. Towards an explanation of household portfolio choice heterogeneity: Nonfinancial income and participation cost structures. NBER Working Paper.

Yum, M., 2018. On the distribution of wealth and employment. Rev. Econ. Dyn. 30, 86–105.