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Nonlinear and time-varying risk premia

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ABSTRACT

Facing the puzzling risk-return trade-off, this paper proposes a new model for risk premia to capture nonlinear and time-varying features under the influence of trading volume. Using high-frequency data for the US stock market in Wharton Research Data Services' Trade and Quote database, our empirical findings suggest a significant nonlinear and time-varying contemporary relationship between return and realized volatility, ranging from positive to negative with an up-down-up pattern, summarized as follows. First, the contemporary relationship is positive on inactive trading days when the trading volume is smaller than usual, in which case traders may face no new information or event uncertainty. Second, the relationship is significantly negative when the trading volume is large on active trading days, in which case traders may be over-confident and behave in a risk-seeking fashion. Third, the risk premium tends toward zero during extremely abnormal trading days. Finally, low and high levels of trading volume have asymmetrical influences on risk premia, with a larger absolute value of risk premia for high levels of trading volume. Furthermore, the nonlinear changing autocorrelation of returns is insignificant from zero on normal trading days and most likely different from zero on abnormal trading days. These results provide explanations for the conflicts between financial theoretic and empirical studies.

1. Introduction

Quantifying the relationship between an asset return and its risk is a fundamental but unanswered issue in finance studies despite being the subject of extensive studies for several decades (Badshah, Frijns, Knif, and Tourani-Rad (2016)). On the one hand, under the market efficiency assumption, most traditional capital asset pricing financial theories, such as the intertemporal capital asset pricing model (Merton et al. (1973), hereafter ICAPM), imply that the relationship should be positive as in French, Schwert, and Stambaugh (1987). On the other hand, it has been commonly recognized that the return and volatility of equity are negatively and asymmetrically related; see Bekaert and Wu (2000) and Badshah et al. (2016). These phenomena have been well documented by leverage effects and feedback effects. Other empirical studies contradict these conclusions with uncertain signs of the relationship between return and risk, the so-called risk-return trade-off, as in Glosten, Jagannathan, and Runkle (1993). In this paper, the nonlinear time-varying contemporary relationship between daily returns and realized volatility (hereafter RV) is investigated under the impact of trading volume, which is commonly used as a proxy for levels of news information flow. This nonlinear contemporaneous daily return-volatility relationship cannot be completely characterized by linear simultaneous or asymmetrical relationships, such as the leverage effect and feedback effect, especially at the daily frequency or higher.

Since the seminal work of Merton et al. (1973), which derives a simplified linear and time-invariant partial positive risk-return

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relationship in the famous ICAPM model, there are many studies to estimate and test this relationship. This risk-return trade-off is so fundamental in financial economics that it could be described as the “first fundamental law of finance”. Unfortunately, empirical studies often yield conflicting results, finding a negative or insignificant return-risk trade-off relationship rather than a positive one, as in Ghysels, Santa-Clara, and Valkanov (2005), Rossi and Timmermann (2015), Liu (2017) and the references therein. Other studies find that the relationship is unstable and time-varying; see, for example, the papers in Nyberg (2012), Kinnunen (2014), and Frazier and Liu (2016) and the references therein. These studies examine the risk-return relations more generally by relaxing the restrictive assumption that these relationships linear and time-invariant, for example, using the Markov-switching specification by Ghysels, Guérin, and Marcellino (2014), considering the asymmetrical relations as responding to the states of an economy and market timing in Wu and Lee (2015), and investigating a variety of possible shapes and potential nonlinearities inherent in return dynamics studied by Frazier and Liu (2016). These mixed and confusing results are usually thought to stem from omitted variable problems or differences of models for return and variance; see Scruggs (1998), Guo and Whitelaw (2006), and Kinnunen (2014), and the references therein. Indeed, after finding that the risk-return relation is considerably time-varying, Brandt and Wang (2010) attributed these conflicts to the limitations of the research design. Most studies assume a constant risk-return relationship over time, which is inconsistent with the understanding that investor preferences change over business cycles. Whitelaw (1994) showed that imposing a constant linear return-risk relation can lead to erroneous inferences because of the unstable risk relation.

Although there are several explanations for and many studies on those above mixed and inclusive results, they are far from sufficient. To the best of our knowledge, the nonlinear and time-varying contemporary return-risk trade-off relationship under the impact of news information flow and endogeneity of volatility is rarely studied in the literature. Facing inclusive results, the asymmetric relationship between return and volatility (risk is usually measured by volatility) has been extensively researched, which are often called leverage effects and feedback effects. The generalized autoregressive conditional heteroskedasticity (GARCH)-type models are often used to study these two effects, such as the models in Brandt and Kang (2004) and Bollerslev, Litvinova, and Tauchen (2006). The GARCH model also describes the linear contemporaneous relationship between return and volatility in its volatility-in-mean equation. However, GARCH-type models are mainly determined by lagged squared returns and lagged variance or other exogenous explanation variables, which are ex ante observable. Harvey (2001) argued that the inclusive result of the relationship between return and volatility not only depends on models but also is affected by exogenous predictors. To avoid relying on exogenous predictors and in a more flexible econometric framework, Brandt and Kang (2004) used a latent vector autoregressive (VAR) model to study contemporaneous and intertemporal relationships between the conditional mean and volatility of stock returns. Actually, this method still does not fully consider endogeneity problems. In our paper, volatility is treated as an endogenous variable directly. At the same time, our motivation also comes from the view that the relevance of the risk-return and autocorrelation can fluctuate with levels of information flow (Kinnunen (2014)). In empirical studies, a first-order autoregressive term is often included in the risk-return specification to account for market inefficiency such as non-synchronous trading (Nelson (1991)) or to test whether the lagged return can help to explain the expected return (Ghysels et al. (2005)). Therefore, the focus in this paper is on the nonlinear and time-varying features of the risk-return trade-off and market efficiency under the influences of levels of information flow, which is believed to be an interest topic and vital in finance research.

Indeed, on the one hand, our focus can be implied by the adaptive markets hypothesis (AMH) of Lo, 2004, which is based on the concept of bounded rationality and evolutionary principles. This hypothesis suggests that market participants adopt satisfactory rather than optimal behaviors through heuristics and an evolutionary process under a permanently changing market environment. Prices reflect both information and the prevailing market ecology. AMH implies that the degree of market efficiency is dynamic and context dependent, and it can change in cyclical fashion with market conditions. The first implication of AMH is that the relation between risk and reward is not stable over time, which means that “the equity risk premium is also time-varying and path-dependent” (Lo (2004)). In this study, it is argued that the level of new information is a key reflection of market conditions. This argument is in line with AMH, which posits that changing market conditions are closely linked to the type and amount of available pricing information and how market participants process and use this information. It seems natural to assume that the survival of market participants and trading strategies depends on the level of new information that should be subsumed in prices.

On the other hand, although economic models usually assume that for positive risk premiums, agents are risk averse or risk neutral, this is not always the case in reality. According to the well-known prospect theory of Kai-Ineman and Tversky (1979), researchers commonly seek risks and tend to overweight outcomes with low probability. Overconfidence is a well-known exception to the rule of risk aversion. The cognitive psychology literature shows that investors are usually overconfident about the precision of their knowledge and behave in an irrational fashion when valuing information. Since the levels of information flow are time varying, it is reasonable to infer that the risk preference of investors is also time varying, which causes risk premiums to change over time under the influence of information flow.

Additionally, trading volume relates to new information. Actually, Easley and O'hara (1992) documented that both the presence and absence of trade may signal the existence of new information. Traders observe and learn from the process of trading. In fact, Jones, Kaul, and Lipson (1994) concluded, “our evidence strongly suggests that the occurrence of transactions per se contains all of the information pertinent to the pricing of securities”. Trading volume is a good proxy for information flows (for details and other introductions, see Section 2.2.3 below).

In other words, it is necessary to relax the assumption of a linear risk-return trade-off and instead consider a nonlinear time-varying relationship under the impact of trading volume. Using high-frequency data from the US stock market, this paper provides new insights into the relationship between return and RV as well as the autocorrelation of returns. Realized volatility has become a common subject of many studies because it is superior and simpler than conventional volatility models such as GARCH and/or stochastic volatility models. RV makes full use of the available intraday information and is less noisy and more informative on the current level of volatility. Various works have explored RV, which can be used in practice; see, for examples, Andersen and Bollerslev (1998), Andersen, Bollerslev, Diebold, and Ebens (2001), Barndorff-Nielsen and Shephard (2002), Andersen, Bollerslev, Diebold, and Labys (2003), and the references therein.

Considering the differences among aggregated market, portfolio and individual stocks, we choose the S&P 500 index (SPX), the SPDR S&P 500 ETF Trust (SPY) and ten large capital companies as our study samples, and then we find strong evidence of a nonlinear and time-varying relationship between return and RV as well as autocorrelation under the impact of trading volume. The relationship can range from positive to negative nonlinearly under the effects of trading volume in a fixed and similar pattern for aggregated market, portfolio and individual stocks.

The main motivation of this study comes from the empirical analysis of the following real example by comparing the model in (Eq. (8)) for the constant regression and the model in (Eq. (9)) for the threshold regression in Section 3. To show our empirical evidence, we plot the estimations of SPX, SPY, Apple Company (AAPL), and Google Company (GOOG) representing the aggregated market, portfolio and individual stocks, respectively, in Figs. 4 and 5, which present the nonlinear and constant (denoted by the green lines) relationships between returns and risk (RV), where the number of trades is used to indicate the trading volume and the proxy for information. Compared with the negative constant risk premium coefficients, these two figures obviously show that the coefficient of the risk-return trade-off is nonlinear and fluctuates with changing trading volume. The detailed results are reported in Section 3.

Our contributions in this paper can be summarized as follows. First, the risk-return relationship is positive on inactive trading days when trading volume is lower than usual. In this case, facing event uncertainty or no news, most investors are risk averse and choose "slow trade" or no trade, requiring a positive premium for risks and liquidity. However, when the trading volume is extremely low, the premium approaches zero, which may indicate the risk-neutral preference of noise traders, who dominate the trading process. As the changes in trading volume increase from negative toward zero, the relationship first increases and then decreases.

Second, the risk-return trade-off decreases to negative values on active trading days when the trading volume is higher than usual. There are three reasons for this phenomenon. First, the increase in trading activities increases the liquidity of stocks, which decreases the risks of inventory. A second reason is the increase in the proportion of informed traders who trade many shares in the direction suggested by their knowledge. Finally, overconfidence leads traders to behave in a risk-seeking fashion, chasing hot stocks and easily overacting. Such traders are likely to be irrational and prefer risk-seeking with negative premiums. However, during extremely active trading days, the relationship trends toward zero after reaching the lowest point because of the different proportions of informed and non-informed participants engaged in speculation and noise trading.

Third, the relationship is approximately zero on normal days or on slightly inactive trading days when the changes in trading volume are approximately zero. During these periods, there is no news, and traders are risk neutral. The main participants are traders who need usual liquidity and risk-neutral noise traders who trade at any time.

Finally, the absolute value of the lowest negative premium is larger than that of the highest positive premium, indicating that the risk-return trade-off is asymmetrically affected by trading volume. In addition, the negative premiums are more significant than positive premiums. This phenomenon is due mainly to multiple effects, such as the increased proportion of informed traders, overconfidence and changes in risk preference during active trading days.

These findings are much more rich and informed than those based on the assumption of a linear relationship between risk and return. Furthermore, we find strong evidence that the autocorrelation or predictability of returns (market inefficiency) is related to trading volume, although there seem to be no significant signs of autocorrelation on normal trading days. The autocorrelation of returns is much more likely to be significantly different from zero on extremely active or inactive trading days, which indicates that the stock market becomes more easily inefficient during abnormal trading days than during normal trading days.

In summary, we find strong and robust evidence that the contemporary relationship between returns and volatilities is nonlinear and time-varying under the impact of changing information flows and the market environment. This relationship has a specific and fixed fluctuating pattern related to trading volume. The autocorrelation or predictability of returns that reflects stock market efficiency is also related to trading volume but with no fixed weaving patterns. Our findings can help explain the inconclusive and mixed results of financial theoretical and empirical studies as well as contradictions among them.

The remainder of this paper is constructed as follows. First, our econometric model and its estimation method are presented in Section 2. Then, we describe the data and report the empirical results in Section 3. Next, some robustness checks for our models are presented in Section 4. Finally, Section 5 concludes the paper.

2. Econometric modeling procedures

2.1. Econometric model

According to the ICAPM as in Ghysels et al. (2005), Nyberg (2012), and Kinnunen (2014), the risk-return relationship can be expressed as follows:

$$\mathrm{E}_{t-1}[r_t] = \mu + \lambda\,\mathrm{Var}_{t-1}[r_t], \tag{1}$$

where $\mathrm{E}_{t-1}[r_t]$ is the conditional expected return on information set Ω_{t-1} and $\mathrm{Var}_{t-1}[r_t]$ is its conditional variance on Ω_{t-1}. Here, λ is the price of asset risk or the coefficient of an investor's risk aversion, which should be positive to indicate a risk premium in ICAPM. According to the theory of ICAPM, μ should be equal to zero in an efficient market. However, a constant μ is often added to represent market imperfection, such as trading costs and taxes or missing factors; see Ghysels et al. (2005) and the references therein. Additionally, a first-order autoregressive term is often included in (Eq. (1)) as

$$\mathrm{E}_{t-1}[r_t] = \mu + \lambda\,\mathrm{Var}_{t-1}[r_t] + \rho\, r_{t-1}. \tag{2}$$

This autoregressive component (ρr_{t-1}) in (Eq. (2)) considers non-synchronous trading as in Nelson (1991) and De Santis et al. (1997) or tests the assumption that the lagged returns can predict future returns as in Bollerslev, Engle, and Wooldridge (1988) and Ghysels et al. (2005). It is well documented that ρ should be zero in an efficient market. However, in empirical studies, this term is often found to be significant; see, for example, De Santis et al. (1997) and Donaldson and Kamstra (2005). Furthermore, according to AMH, the degree of market efficiency is dynamic and context dependent, and it can change in cyclical fashion with market conditions. To capture the dynamic and context-dependent characteristics, Kinnunen (2014) studied the following model:

$$\mathrm{E}_{t-1}[r_t] = \mu + \varphi_{t-1}\lambda\,\mathrm{Var}_{t-1}[r_t] + (1-\varphi_{t-1})\rho\, r_{t-1}, \tag{3}$$

where $\varphi_{t-1} \in [0,1]$ is a time-varying weight of the risk-return trade-off, which can indicate the degree of market efficiency and investors' rationality. The closer φ_{t-1} is to one, the more efficient the market is and the more rationally investors behave. As φ_{t-1} approaches zero, prices reflect information only partially, and market irrationality dominates, indicating market inefficiency. Therefore, φ_{t-1} captures the information-related features of the risk-return trade-off and market efficiency. However, the problem with the empirical application of this model is its assumption on φ_{t-1}, which is a logistic function of information variables: $\varphi_{t-1} = [1 + \exp(\beta' S_{t-1})]^{-1}$, β is a vector of parameters and S_{t-1} is a vector of predetermined variables that serve as proxies for or are related to the level of information flow. Although a logistic function can guarantee $\varphi_{t-1} \in [0,1]$, we argue that the assumption of a monotonic logistic function may be too restrictive for the time-changing weight coefficient and its implied mechanism, which may produce unreliable results. Since it is very difficult to specify a fixed function of time-varying weight, we adopt a functional-coefficient model to allow some coefficients of the model to be unknown functions of other variables; see, for example, the paper by Cai, Fan, and Yao (2000) for details. Not only can the functional-coefficient model capture nonlinearity and heterogeneity, but it can also accommodate structural information, as argued in Cai (2010). The functional-coefficient model can be expressed as follows:

$$\mathrm{E}_{t-1}[r_t] = \mu + \alpha(u_t)\mathrm{Var}_{t-1}[r_t] + \rho(u_t) r_{t-1}, \tag{4}$$

where u_t is a proxy for levels of new information. Here, both $\alpha(\cdot)$ and $\rho(\cdot)$, which capture the unknown nonlinear time-varying characteristic of risk-return trade-off and market efficiency under the influences of news information flow, are unknown coefficient functions of variable u_t. These two coefficient functions are more general than φ_{t-1} in (Eq. (3)) in chasing the dynamic changing environment of the market. Note that the constant term μ does not depend on u_t and reflects the average effect of intrinsic market imperfection (for example, taxes and transaction costs) and other missing factors. This constant term is often included in many empirical studies with statistically significant estimates, although it is not theoretically justified in ICAPM; see Nyberg (2012) and Kinnunen (2014), among others.

Previous studies have generally addressed $\mathrm{Var}_{t-1}[r_t]$ in two ways. The first way is to model its dynamics by exogenous predictors or stochastic processes, such as the GARCH-type models, which use the lagged squared returns or lagged variance or other variables as predictors (see Section 1 for more details). However, different dynamic models of conditional volatility can produce inconclusive and mixed relationships between return and volatility; see the studies by Harvey (2001) and Brandt and Kang (2004). Another way to address $\mathrm{Var}_{t-1}[r_t]$ is to use direct measurements, for example, through absolute returns and range returns. As high-frequency data become increasingly available, superior measures of RV can be used to directly estimate volatility.

Although few empirical studies consider the endogeneity of the conditional variance, $\mathrm{Var}_{t-1}[r_t]$, we argue that the variance $\mathrm{Var}_{t-1}[r_t]$ is an endogenous variable in this paper for the following reasons. First, the conditional return $E_{t-1}[r_t]$ and conditional variance $\mathrm{Var}_{t-1}[r_t]$ are both conditional variables of the information set Ω_{t-1} at time $t-1$. This phenomenon is consistent with the definition of endogenous variables. An endogenous variable in an econometric model is changed or determined by its relationship with other variables within the model and is synonymous with a dependent variable. Second, there are two types of well-known empirical effects between conditional return $E_{t-1}[r_t]$ and conditional variance $\mathrm{Var}_{t-1}[r_t]$, namely, leverage effects and feedback effects, which indicate that the two depend on each other. Finally, if using the superior measures of RV by intradaily data, it might be reasonable to infer that the daily returns and RV have some intrinsic internal relation because of their similar intradaily returns data. Therefore, instead of treating $\mathrm{Var}_{t-1}[r_t]$ as a conditional variable that depends on the variables at time $t-1$ or a latent process variable such as GARCH-type models, it is also assumed that it is an endogenous variable, which can be measured by RV at time t in our paper. In this way, we can not only avoid specifying fixed dynamic coefficient functions of volatility and autocorrelation as well as the problem of endogeneity but also improve the model using this superior nonparametric estimator. For the proxy variable for the information flow u_t, it is assumed that it is an exogenous variable. Our arguments are as follows. According to the AMH of Lo (2004), changing market conditions are closely linked to the type and amount of available pricing information and how market participants process and use this information. It is reasonable to assume that the survival of market participants and trading strategies depends on the level of new information flow, which is also stated in Kinnunen (2014). That is, new information flow reflects the changing market conditions, while prices reflect the information flow and market conditions. Namely, information flow affects prices, and prices depend on information flow. Thus, the empirical model can be expressed in the following form:

$$r_t = \mu + \alpha(u_t) RV_t + \rho(u_t) r_{t-1} + \varepsilon_t, \tag{5}$$

where r_t is the daily return, RV_t is the realized volatility, and u_t is the change in trading volume, which is a proxy for the news information flow. The estimation of this model naturally falls into the framework of a partially varying coefficient model with endogenous regressors proposed in Cai, Fang, Lin, and Su (2019), which is particularly discussed in Section 2.3 in detail. The definitions of variables in (Eq. (5)) and model estimations are presented in the following subsections.

2.2. Variable definitions and computations

2.2.1. Returns

Define the logarithm return in this paper as the return by following the conventional definition in financial studies, especially under the stochastic process framework. If the transaction price P_t denotes the stock prices at time t, then the return r_t is defined as follows:

$$r_t = \log(P_t) - \log(P_{t-1}), \quad \mathrm{t} = 2, 3, \cdots \mathrm{T}.$$

2.2.2. Measure of volatility

Andersen and Bollerslev (1998) first proposed RV as a measure of integrated variance. With a continuous-time stochastic process for the log-price, the intraday return $r_{t,\ j}$ is defined as the j-th return in day t. Then, RV is defined by

$$RV_{\mathrm{t}} = \sum_{j=1}^{M} r_{t,j}^2, \quad \mathrm{t} = 1, 2, \cdots \mathrm{T},$$

where M is the number of sampling intervals of each day. It is well known in the literature that RV converges in probability to the integrated variance as the sampling interval becomes small enough ($M \to \infty$) under some regularity conditions. Indeed, Andersen et al. (2001) and Andersen et al. (2003) showed that

$$RV_{\mathrm{t}} \to \int_0^t \sigma_s^2 d_s + \sum_{0<s\le t} (\Delta p_s)^2, \quad \text{as M} \quad \to \infty\ ,$$

where $\Delta p_s = p_s - p_{s-}$ captures a jump, if present. Therefore, RV can be arbitrarily close to the true variance as the sampling frequency increases, but RV can also be seriously biased because of microstructure noise, such as price discreteness, non-synchronous trading and bid-ask bounce. In fact, microstructure noise can dominate RV at an ultrahigh frequency, in which case RV does not present the true variance at all. Therefore, there are many studies that trade off bias and convergence through optimal sampling intervals by sparse sampling or other techniques; see, for example, Aït-Sahalia, Mykland, and Zhang (2005), Andersen, Bollerslev, and Meddahi (2005), Zhang, Mykland, and Aït-Sahalia (2005), and Jacod, Li, Mykland, Podolskij, and Vetter (2009). In this paper, by following the commonly used sparse sampling method, a 5-min sampling interval is used.

2.2.3. Trading volume as a proxy for the level of information

According to the saying “it takes volume to make prices move”, there must be some sellers and some buyers to make a transaction at a given price in a liquid market with a large number of traders. According to Easley and O'hara (1992), “traders learn from both trades and the lack of trades because each may be correlated with different aspects of information”. Thus, trading volume could proxy for information because price fluctuations are related to buying and selling pressures. Buyers and sellers make decisions according to public or private information and the trading process itself, which makes prices fluctuate and, in turn, affects the trading volume. Uninformed and informed market participators trade their financial assets based on their own information. It is impossible for the financial asset price to vary without any trading activities. Therefore, the trading process must induce trading volume. Trading volume is often used to proxy for information flow, even in the current information age. For a review, the reader is referred to the paper by Queirós (2016). Indeed, Jones et al. (1994) concluded that “our evidence strongly suggests that the occurrence of transactions per se contains all of the information pertinent to the pricing of securities”.

There are several measures of trading volume, such as share volume (Gallant, Rossi, and Tauchen (1992); Andersen (1996)), dollar volume (Lakonishok and Vermaelen (1986)), turnover (Chae (2005)), and total number of trades (Chan and Fong (2000)). Among these measures, the daily number of trades and daily turnover are the most common and reliable two measurements. The strategic asymmetric information models suggest that informed investors fragment their trades into smaller and medium-sized trades to camouflage their intent and thus benefit from their private information, which makes the number of trades more informative than other measures; see, for example, Chordia and Subrahmanyam (2004) and Giot, Laurent, and Petitjean (2010). Considering the large difference in magnitude between daily returns and the number of trades, we use the differences between the logarithm number of trades and its average during a period of trading days, given by

$$LogTR\_NUM_t = \log(TR\_NUM_t) - \frac{1}{K}\sum_{i=-K}^{-1} \log(TR\_NUM_{t+i}), \tag{6}$$

where TR_NUM_t denotes the daily number of trades. Lo and Wang (2000) studied the turnover of individual stocks and theoretically justified the use of turnover as a measure of trading volume. The daily turnover of a stock is defined as the total number of shares traded that day divided by the total number of shares outstanding. Usually, daily turnover is nonstationary. To overcome nonstationarity, take the logarithm transformation, followed by de-trending, as in most studies such as Llorente, Michaely, Saar, and Wang (2002). In addition, a tiny constant value ($c_0 = 0.00000255$) is added to the turnover before taking the logarithm, which avoids zero value of daily turnover, as in Richardson, Sefcik, and Thompson (1986), given by

$$LogTurnover_t = \log(Turnover_t + c_0) - \frac{1}{K}\sum_{i=-K}^{-1}\log(Turnover_{t+i} + c_0), \tag{7}$$

where $Turnover_t$ denotes the daily turnover on day t. Based on our empirical data, $K = 22$ is used, which is the number of trading days in 1 month, and abnormal trading days are defined when trading volume is much higher than the average trading volume after de-trending.

2.3. Estimation procedure

In this paper, the estimation procedure comes from a partially varying coefficient model with endogenous regressors proposed by Cai et al. (2019). Building on a vast amount of literature on nonparametric estimation of instrumental variables models, Cai et al. (2019) proposed a semiparametric functional-coefficient instrumental variables model (See (1) in Cai et al. (2019)). This model is very general and includes many popular models, such as the nonparametric functional-coefficient model by Cai, Das, Xiong, and Wu (2006), the model with a univariate discrete endogenous regressor by Das (2005), the threshold instrumental variables model by Caner and Hansen (2004) and semiparametric functional-coefficient models studied in the literature.

Estimating the model in (Eq. (5)) efficiently is not an easy task. To avoid the so-called curse of dimensionality, Cai et al. (2019) proposed using linear projection conditional on the smoothing variables. To remove the effects of residuals from the reduced-form equation when the variation in the reduced form equation is large or the structural equation and the reduced form equation have positive correlated variations, they also developed a novel modified approach of the profile least-squares method to estimate the constant coefficients. To this end, a three-stage approach is needed to estimate the functions and parameters in the model in (Eq. (5)), described below in detail. Note that in this paper, RV_{t-1} is chosen as the instrumental variable of RV_t. Many previous papers conclude that RV has a strong persistent and long memory; see Andersen et al. (2003) and Bandi and Perron (2006). As a result, the lagged variable of RV is a sensible and natural choice for the instrumental variable.

First, it constructs a projection of endogenous variables on a set of instrumental variables. Let Z_t include the constant term, the exogenous variable r_{t-1} and instrument variable RV_{t-1}. Then, the linear projection of (RV_t, r_{t-1}) on Z_t conditional on u_t, denoted by $(\widehat{RV}_t, \hat{r}_{t-1})$, where

$$\widehat{RV}_t^T = (Z_t^T, \mathbf{0}_{1\times q})[\mathbf{D}_Z(u_t)^T\mathbf{H}(u_t)\mathbf{D}_Z(u_t)]^{-1}\mathbf{D}_Z(u_t)^T\mathbf{H}(u_t)\mathbf{RV},$$

and

$$\hat{r}_t^T = (Z_t^T, \mathbf{0}_{1\times q})[\mathbf{D}_Z(u_t)^T\mathbf{H}(u_t)\mathbf{D}_Z(u_t)]^{-1}\mathbf{D}_Z(u_t)^T\mathbf{H}(u_t)\mathbf{r},$$

respectively, with $\boldsymbol{H}(u) = \text{diag}\,(K_h(u_2 - u), \cdots, K_h(u_n - u))$, $\boldsymbol{RV} = (RV_2, \cdots, RV_n)^T$, $\boldsymbol{r} = (r_1, \cdots, r_{n-1})^T$, $\boldsymbol{D}_{Z(u)} = \begin{pmatrix} Z_2^T & \frac{u_2 - u}{h}Z_2^T \\ \vdots & \vdots \\ Z_n^T & \frac{u_n - u}{h}Z_n^T \end{pmatrix}$ and

$$\boldsymbol{D}_{X(u)} = \begin{pmatrix} RV_2^T & \frac{u_1 - u}{h}RV_2^T \\ \vdots & \vdots \\ RV_n^T & \frac{u_n - u}{h}RV_n^T \end{pmatrix}.$$

Second, it estimates the constant coefficients by the profile least squares approach. Based on Cai et al. (2019), a modified profile least squares estimator of β is given by

$$\tilde{\beta} = [\hat{\mathbf{r}}^T(\mathbf{I}_n - \hat{\mathbf{S}})^T(\mathbf{I}_n - \tilde{\mathbf{S}})\mathbf{r}]^{-1}\hat{\mathbf{r}}^T(\mathbf{I}_n - \hat{\mathbf{S}})^T(\mathbf{I}_n - \tilde{\mathbf{S}})\mathbf{Y},$$

where $\boldsymbol{I}_n$ is the $(n-1) \times (n-1)$ identity matrix, $\boldsymbol{Y} = (r_2, \cdots, r_n)^T$,

$$\hat{\boldsymbol{S}} = \begin{pmatrix} (\widehat{RV}_2^T, \mathbf{0}_{1\times p})[\widehat{\mathbf{D}}_X(u_2)^T\mathbf{H}(u_2)\widehat{\mathbf{D}}_X(u_2)]^{-1}\widehat{\mathbf{D}}_X(u_2)^T\mathbf{H}(u_2) \\ \vdots \\ (\widehat{RV}_n^T, \mathbf{0}_{1\times p})[\widehat{\mathbf{D}}_X(u_n)^T\mathbf{H}(u_n)\widehat{\mathbf{D}}_X(u_n)]^{-1}\widehat{\mathbf{D}}_X(u_n)^T\mathbf{H}(u_n) \end{pmatrix},$$

$$\widehat{\mathbf{D}}_X^T(u) = \mathbf{D}_Z(u)[\mathbf{D}_Z(u)^T\mathbf{H}(u)\mathbf{D}_Z(u)]^{-1}\mathbf{D}_Z(u)^T\mathbf{H}(u)\mathbf{D}_X(u),$$

and

$$\tilde{\mathbf{S}} = \begin{pmatrix} (RV_2^T, \mathbf{0}_{1\times p})[\widehat{\mathbf{D}}_X(u_2)^T\mathbf{H}(u_2)\widehat{\mathbf{D}}_X(u_2)]^{-1}\widehat{\mathbf{D}}_X(u_2)^T\mathbf{H}(u_2) \\ \vdots \\ (RV_n^T, \mathbf{0}_{1\times p})[\widehat{\mathbf{D}}_X(u_n)^T\mathbf{H}(u_n)\widehat{\mathbf{D}}_X(u_n)]^{-1}\widehat{\mathbf{D}}_X(u_n)^T\mathbf{H}(u_n) \end{pmatrix}.$$

Note that as shown in Cai et al. (2019), this modified estimator is more efficient than the conventional profile least squares to remove the effects of residues from the reduced form in their simulation study and empirically relevant cases.

Third, it estimates the functional coefficients using the kernel method at given value u,

$$\begin{pmatrix} \widetilde{A}(u) \\ h\widetilde{A}'(u) \end{pmatrix} = [\widehat{\mathbf{D}}_X^T(u)\mathbf{H}(u)\widehat{\mathbf{D}}_X^T(u)]^{-1}\widehat{\mathbf{D}}_X^T(u)\mathbf{H}(u)(\mathbf{Y} - \mathbf{r}\widetilde{\beta}).$$

By Cai et al. (2019), the bias term of estimators of functional coefficients $\widetilde{A}(u)$ is of order h^2 and is the same as the bias in the semiparametric functional-coefficient model and in a nonparametric functional-coefficient instrumental variables model. Furthermore, they established the consistency and asymptotic normality of these proposed estimators. This semiparametric framework and the functional-coefficient setup can not only alleviate the curse of dimensionality in a multivariate regression framework but also avoid the so-called ill-posed inverse problem in general nonparametric instrumental variables models. For the challenging problem of selecting bandwidth, the idea in Cai et al. (2019) is used. For the first two stages, cross-validation is employed to select the bandwidth in the order $h_1 = Cn^{-1/5}$ first, and then $h_2 = Cn^{-3/8}h_1$ is chosen as the bandwidth. For the choice of bandwidth in the third stage, the cross-validation selection criterion is adopted again. For details of the estimation procedure, the reader is referred to the paper by Cai et al. (2019).

3. Empirical results

3.1. Data and sample description

We choose the S&P 500 Index (SPX) and SPDR S&P 500 ETF Trust (SPY) as representative for aggregated markets and portfolio performance. We also select ten large total capital individual stocks from the American stock markets, namely, Amazon (AMZN), Apple, Google, Intel (INTC), JPMorgan Chase (JPM), Microsoft (MSFT), AT&T (T), Walmart (WMT), Johnson & Johnson (JNJ), and Exxon Mobil C (XOM). The uppercase letters in the brackets behind each company name are the Sym_Root, which identify the stocks in the Center for Research in Security Prices database. We choose 10-year period data ranging from January 2, 2009, to December 31, 2018. We construct the variables using tick-by-tick data from the Trade and Quote database (TAQ) from Wharton Research Data Services. To address the microstructure noise, we extract 5-min price data from the database to construct RV. As usual, the most recent trading price at each point is used. The number of trades and trading shares is the sum of all trades during the trading time. All of the data occurred in regular trading hours from 9:30:00 to 16:00:00. After obtaining the data, the dataset is cleaned according to some procedures used by Holden and Jacobsen (2014) for the daily TAQ. However, since the library of the Oxford-Man Institute of Quantitative Finance provides RV of the value-weighted return of SPX, we directly download it for SPX. The trading volumes are constructed by following Lo and Wang (2000). According to their study, we use the equal averaging turnovers, the aggregated sum of the number of trades of all listed S&P 500 individual stocks as the *turnover* and *TR_NUM* of SPX, respectively, for each day. The list of firms and the outstanding shares of each firm listed on the SPX come from CRPS.

Because all of the data used to calculate the variables concern only trading hours, overnight returns are excluded. This avoids the need to adjust prices for splits or dividends. Therefore, the closing-to-opening daily returns are used as in Chan and Fong (2006) and

Table 1
Descriptive statistics of daily closing-to-opening returns, RV, the number of trades and turnover.

	mean	min	max	Std.	Skewness	Kurtosis	ρ_1	sig.
Daily close-to-open return								
SPX	0.0002	−0.0495	0.0447	0.0086	−0.2701	7.0982	−0.0536	0.0072
SPY	0.0002	−0.0454	0.0440	0.0081	−0.3041	7.5141	−0.0654	0.0010
AAPL	0.0000	−0.0684	0.0832	0.0135	0.0222	4.9138	−0.0715	0.0003
GOOG	−0.0001	−0.0841	0.0485	0.0123	−0.4690	6.1841	−0.0125	0.5301
Daily realized volatility								
SPX	0.0001	0.0000	0.0037	0.0002	8.3470	142.1751	0.6018	0.0000
SPY	0.0001	0.0000	0.0024	0.0001	8.8001	137.8598	0.6366	0.0000
AAPL	0.0002	0.0000	0.0059	0.0003	11.2245	220.8345	0.5427	0.0000
GOOG	0.0002	0.0000	0.0049	0.0002	7.7622	111.5773	0.5388	0.0000
Daily number of trades								
SPX	11,895,170.61	2,983,057	30,305,798	3,200,912.82	1.4378	6.7274	0.8324	0.0000
SPY	378,508.28	82,521	2,279,628	213,216.21	2.1613	10.2665	0.8225	0.0000
AAPL	156,115.86	30,129	892,023	94,756.02	1.9524	9.4470	0.7788	0.0000
GOOG	24,198.48	5602	121,577	13,027.53	2.1377	9.9371	0.7013	0.0000
Daily turnover								
SPX	0.9910	0.2958	2.5910	0.3356	1.4512	5.6857	0.8965	0.0000
SPY	0.1590	0.0259	0.9555	0.1107	1.8489	8.6774	0.8641	0.0000
AAPL	0.0132	0.0023	0.0713	0.0090	1.5608	6.7421	0.8195	0.0000
GOOG	0.0073	0.0021	0.0462	0.0040	3.0294	18.9817	0.6057	0.0000

The 8–9th columns show the first-order autocorrelations and corresponding significance probabilities, respectively.

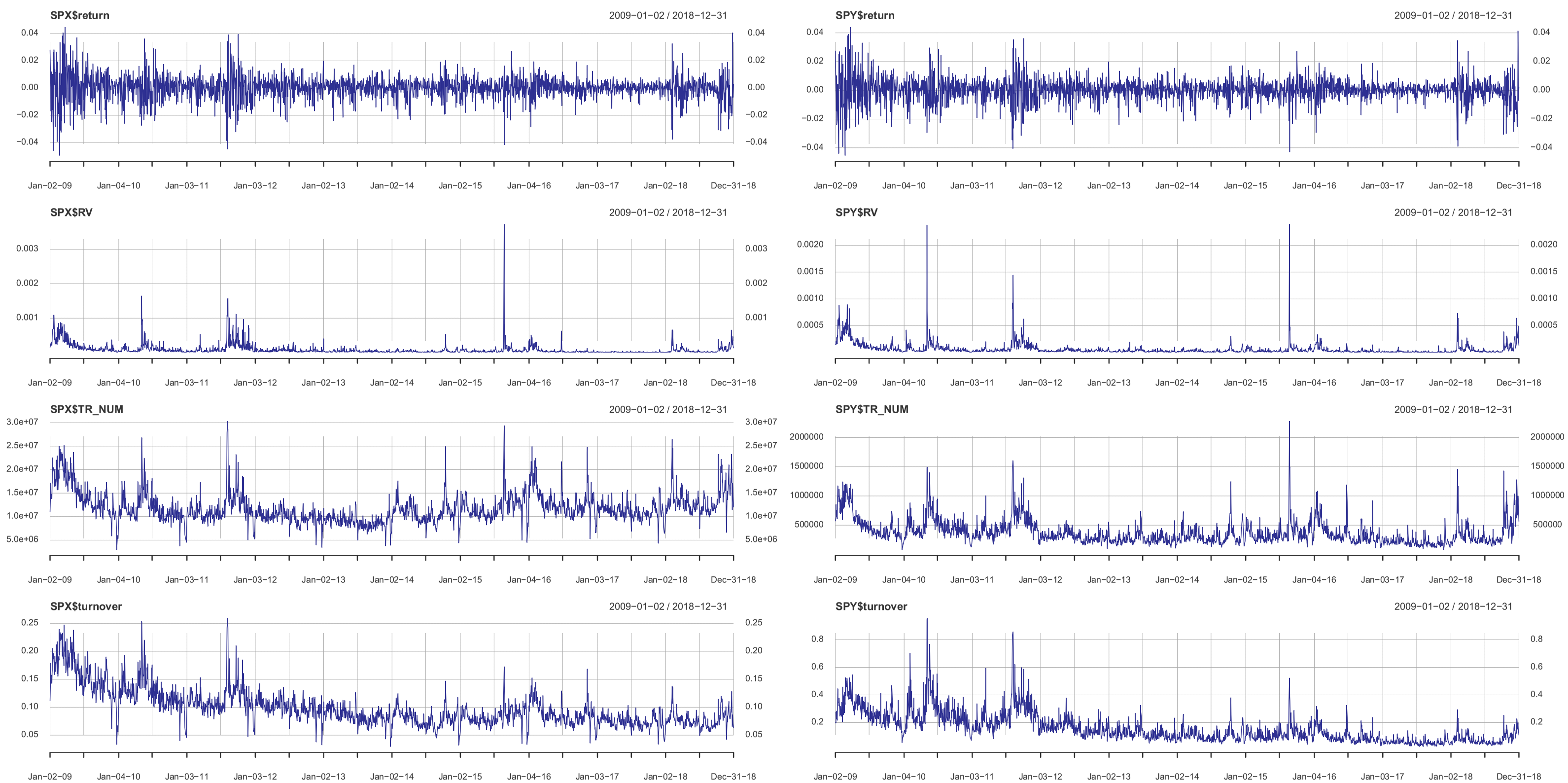

Fig. 1. The series of closing-to-opening daily returns, RV, *TR_NUM* and Turnover. The left subfigures are series of SPX, and those on the right are SPY series. The first line is the series of returns, the second line is RV, the third is *TR_NUM* and the fourth is *Turnover*.

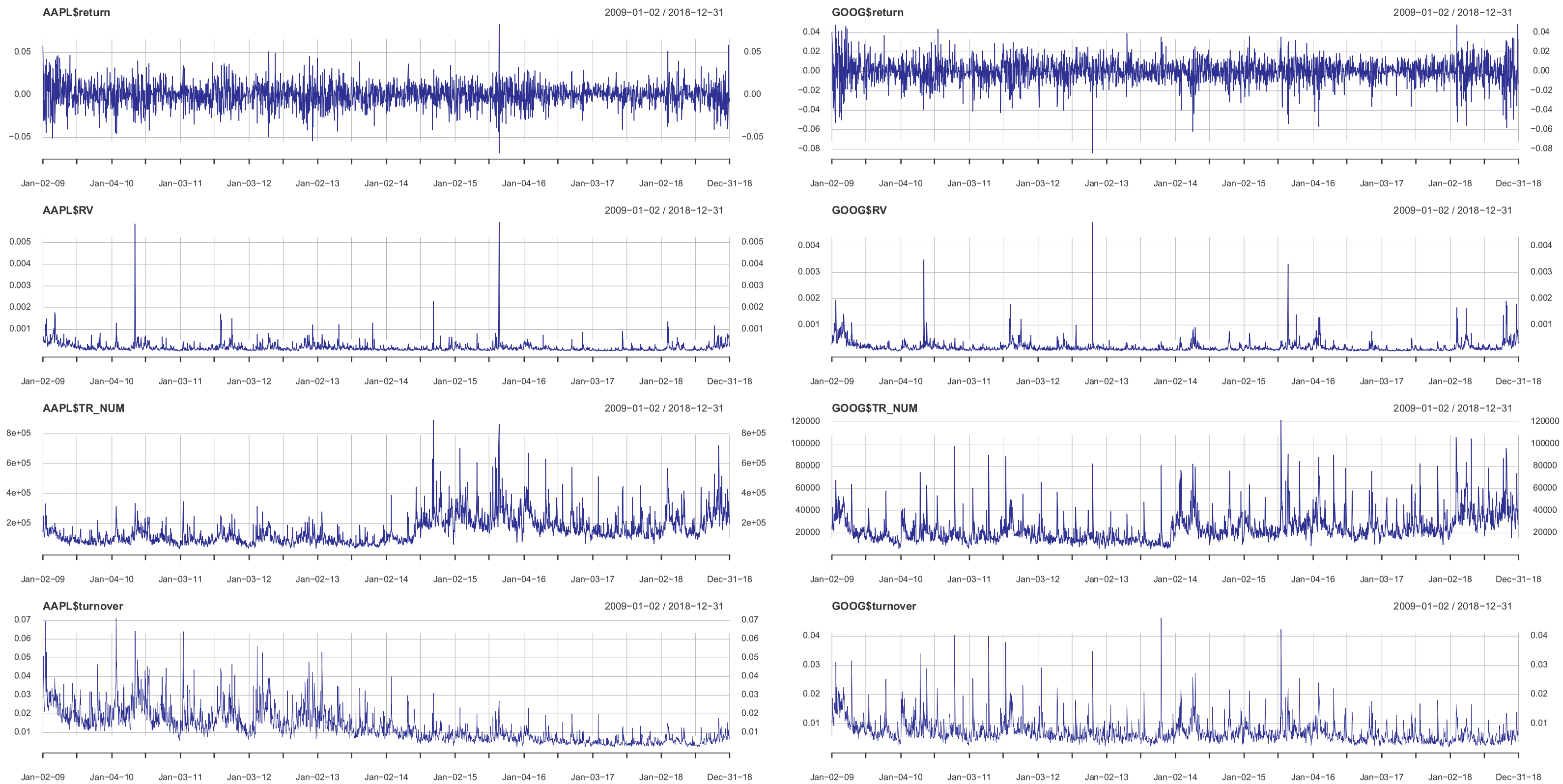

Fig. 2. The series plots of AAPL and GOOG. The layouts are the same as those in Fig. 1. The left subfigures are series of AAPL, and those on the right are GOOG's series. The reverse fluctuations of AAPL between *TR_NUM* and *Turnover* are abnormal due to its dramatic increase of outstanding shares on June 9, 2014 by 7 times.

Patton and Sheppard (2015). The opening price is the first trade price, and the closing price is the last trade price of the day. The opening and closing price of SPX is the simple averaging opening and closing prices of all S&P 500 stocks. Because of similar fluctuating curves of risk premiums for the selected samples, to show our findings clearer and more simplicity, we report the results of SPX, SPY, AAPL and GOOG in this paper as illustrations for aggregated markets, portfolios and individuals. Other results are available upon request. Table 1 describes the statistics for closing-to-opening daily returns, RV, the number of trades and turnover.

From Table 1, one can see that the kurtosis values of all returns are much larger than three, indicating a non-Gaussian distribution, a typical style feature of finance returns. Another important feature to be noted is the ambiguous significance of the first-order autocorrelation of daily returns, with five firms (AMZN, GOOG, JNJ, T and XOM) having insignificant autocorrelations, which are not shown in this table. For RV, all firms have large autocorrelation with statistical significance and a non-Gaussian distribution. In fact, previous studies show that long memory is an inherent attribute of RV; see, for example, Bandi and Perron (2006) and Corsi (2009) for details. The daily number of trades and turnover are not only non-Gaussian distributions but also highly significant autocorrelations, namely, long-memory features. It is easy to see that *TR_NUM* and *Turnover* have strong autocorrelations for several lags, which is in line with previous studies, such as Bollerslev and Jubinski (1999) and Fleming and Kirby (2011). Using fractionally integrated time-series models and high-frequency data, Fleming and Kirby (2011) found clear evidence that volume and volatility both express long-memory behavior.

To show the inherent features of trading volume, we plot the series of the number of trades and turnover in Figs. 1 and 2, respectively, instead of *LogTR_NUM* and *LogTurnover*. Usually, there are fluctuation tendencies of the same ups and downs for the RV, *TR_NUM* and *Turnover* series during the period of volatility, namely, the cross-relationships among them are significantly positive in general. The reverse fluctuations of APPL in *TR_NUM* and *Turnover* in Fig. 2 during the latter half of the period are abnormal. In fact, AAPL had a dramatic increase of outstanding shares during the sample period by 7 times from 861,381,000 to 6,029,667,000 on Jun 9, 2014, and then gradually decreased in the following period, which had unequal effects on its numbers of trades and turnovers, resulting in a dramatic increase in the number of trades, while the opposite was true for turnover. See the right third and fourth subfigures in Fig. 2. There are two important conclusions to be noted here. First, the assumption of a significant constant positive linear relationship between daily returns and RV is unreasonable. Fig. 3 displays the scatters of return and RV. Although returns diffuse as the RV increases, there is no obvious indication of one-directional diffusion. Fig. 3 visually indicates that a larger RV accompanies larger absolute returns. Even though the correlation between return and RV is significantly negative, as shown in Table 2, it is very small. In fact, not all correlations are significant for individual stocks; for example, the coefficients for INTC and JNJ are insignificant during our sample period. Table 2 describes the Pearson correlation coefficients matrix. Second, the cross-relationships between *TR_NUM* and *Turnover* are highly positive. Based on the selected samples, our results vary from 0.515 to 0.969, except for AAPL. The abnormal phenomenon of AAPL is due to a dramatic increase in outstanding shares. The high relevance between *TR_NUM* and *Turnover* is consistent with the fact that both *TR_NUM* and *Turnover* are good measures of trading volume.

3.2. Modeling results

3.2.1. Results for the parametric models

As illustrated in Section 1, to argue that the nonlinear and time-varying coefficient indeed exists, the following two regressions are investigated. The first regression model is the constant regression as

$$r_t = \mu + \alpha_c RV_t + \rho_c r_{t-1} + \varepsilon_t, \tag{8}$$

and the second model is the following threshold regression:

$$r_t = \mu + \alpha_0 RV_t I(u_t \geq 0) + \alpha_1 RV_t I(u_t < 0) + \rho_c r_{t-1} + \varepsilon_t, \tag{9}$$

where α_c is the constant risk premium, u_t denotes the level of new information flow, which is agented by *LogTR_NUM*(Eq. (6)) or *LogTurnover*(Eq. (7)) in this paper. α_0 is the risk premium when $u_t \geq 0$, and α_1 is the risk premium when $u_t < 0$. We plot these regression coefficients in the models in (Eq. (8)) and model (Eq. (9)) in Figs. 4, 5, 7 and 8 (denoted in green lines), respectively, by the ordinary least squares method, which clearly implies that α_0 and α_1 are not the same and present nonlinearity. Taking SPX as an example, although the constant coefficient α_c is significantly less than zero overall, α_1 is significantly larger than zero. Conversely, α_0 is less than zero. Furthermore, we also estimate the empirical application of (Eq. (3)) using the smooth transition autoregressive (STAR) model framework, and the results for SPX and SPY are given in Appendix, where one can see that the risk premium is a declining and asymmetric logistic curve. However, it is argued that this binary or predefined logistic function nonlinearity is too rough or inflexible to map the full graph of dynamic nonlinear features of risk premiums. We have to study the "real" effects of trading volume on risk premium in a more general framework. A good method to do so is the functional-coefficient regression model. Functional-coefficient models have many advantages, such as depicting the finer structure of the underlying dynamics; see Cai et al. (2000). It also does not need to assume a fixed function of coefficient and is more flexible. In other words, nonlinear and time-varying coefficients actually exist, which motivates us to further consider the proposed model in (Eq. (4)).

3.2.2. Results for the number of trades as trading volume

Figs. 4 and 5 plot the main estimations and results of SPX, SPY, AAPL and GOOG. The results for other samples have similar weaving patterns of risk premium, which are available upon request. Figs. 4 and 5 clearly show that the nonlinear relationship between returns and risk (RV) exists and fluctuates with changing trading volume, where the number of trades is used to indicate the

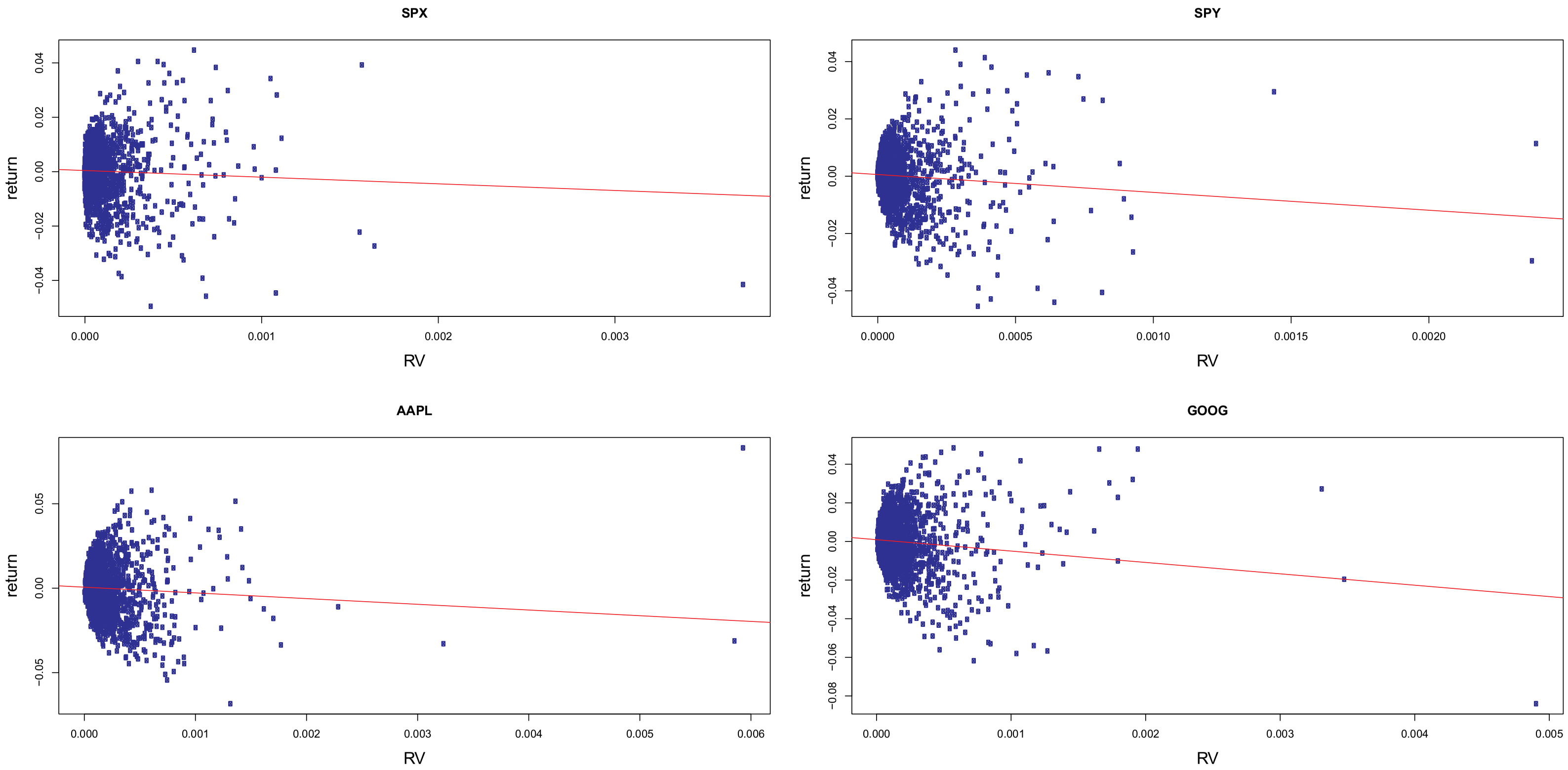

Fig. 3. Scatter plots of the returns and RV. The red line represents the tendency. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 2
The Pearson correlation coefficients matrix.

SYM_ROOT	Variables	return	RV	Turnover	TR_NUM
SPX	return	1.000			
	RV	−0.044			
		(−2.203)			
	Turnover	−0.037	0.565	1.000	
		(−1.857)	(34.354)		
	TR_NUM	−0.101	0.579	0.709	1.000
		(−5.0878)	(35.563)	(50.415)	
SPY	return	1.000			
	RV	−0.090	1.000		
		(−4.550)			
	Turnover	−0.097	0.666	1.000	
		(−4.903)	(−44.779)		
	TR_NUM	−0.137	0.809	0.832	1.000
		(−6.933)	(−69.118)	(−75.152)	
AAPL	return	1.000			
	RV	−0.064	1.000		
		(−3.188)			
	Turnover	−0.052	0.506	1.000	
		(−2.619)	(29.392)		
	TR_NUM	−0.072	0.372	−0.014	1.000
		(−3.621)	(−20.114)	(−0.697)	
GOOG	return	1.000			
	RV	−0.111	1.000		
		(−5.594)			
	Turnover	−0.093	0.542	1.000	
		(−4.702)	(32.345)		
	TR_NUM	−0.124	0.539	0.677	1.000
		(−6.286)	(32.053)	(46.094)	

The numbers in brackets are corresponding T-statistics. All of the coefficients are significant expect one between *TR_NUM* and *Turnover* of AAPL. APPL has a significant increase in outstanding shares on Jun 9, 2014, from 861,381,000 to 6,029,667,000, an increase of seven times. Therefore, the trading activity increases after this day, but the turnover decreases. In the following period, the outstanding shares of AAPL gradually decrease. As a result, its correlation coefficient between *TR_NUM* and *Turnover* is insignificant.

trading volume and proxy for information. The risk-premium coefficients (denoted by the blue lines) in Figs. 4 and 5 suggest the following findings.

First, the premium is positive when *LogTR_NUM* is negative, presenting an up-first and down-then pattern, especially for SPX and SPY. In this stage, the premium first increases as *LogTR_NUM* increases. Then, it decreases when *LogTR_NUM* approaches zero. According to the sequential model with event uncertainty studied by Easley and O'hara (1992) and Easley, Kiefer, and O'Hara (1997), the absence of trade suggests a decreased likelihood of an information event or event uncertainty. Facing event uncertainty or no news, most investors are risk averse and choose "slow trade" or no trade, requiring a positive premium for risks and liquidity. The activities of informed traders decrease as the proportion of informed traders decreases; these traders always trade in the direction of their knowledge when events occur. However, the trading activities of uninformed or noise traders are the same regardless of whether events occur. As a result, we can interpret the positive risk-return trade-off to indicate risk aversion during inactive trading days when traders face event uncertainty and the proportion of uninformed traders is high.

Second, the premium is negative during the active trading days when *LogTR_NUM* is positive. According to the sequential model and the conclusion of Jones et al. (1994), the number of trades is a good proxy for information. The increase in trading activity results mainly from the activity of informed traders who trade in the direction of their private information or knowledge. Traders are often overconfident about their knowledge. Overconfidence can increase trading volume and market depth and decrease expected utility; see Odean (1998). People are likely to overreact based on their information and behave in a risk-seeking fashion. Therefore, the premium is negative during these active trading days. However, there is an increasing trend toward zero at the right end of each subfigure when the trading days are extremely active. These trading days are abnormal, and the extremely high number of trades may make it difficult for traders to evaluate information and increase uncertainty. Therefore, the premium increases slowly during these abnormal days.

Third, the premium responds asymmetrically to inactive and active trading days. The peak absolute value of the negative premium on active trading days is larger than that on inactive trading days. It is believed that this comes from the twofold influence of informed traders and overconfidence. On active trading days, on the one hand, informed traders sell or buy stocks based on their knowledge and engage in risk-seeking; on the other hand, the likelihood of overconfidence results in higher risk. This asymmetry can easily produce empirical results of mixed or confused risk premium coefficients if the study samples include unequal numbers of inactive and active trading days.

Finally, the autocorrelation of returns is much more complex and mixed. Fig. 6 plots the autocorrelation $\rho(\cdot)$ in the model (Eq. (5)). From the perspective of the aggregate market (SPX) or portfolio (SPY), the autocorrelation of return presents a decreasing

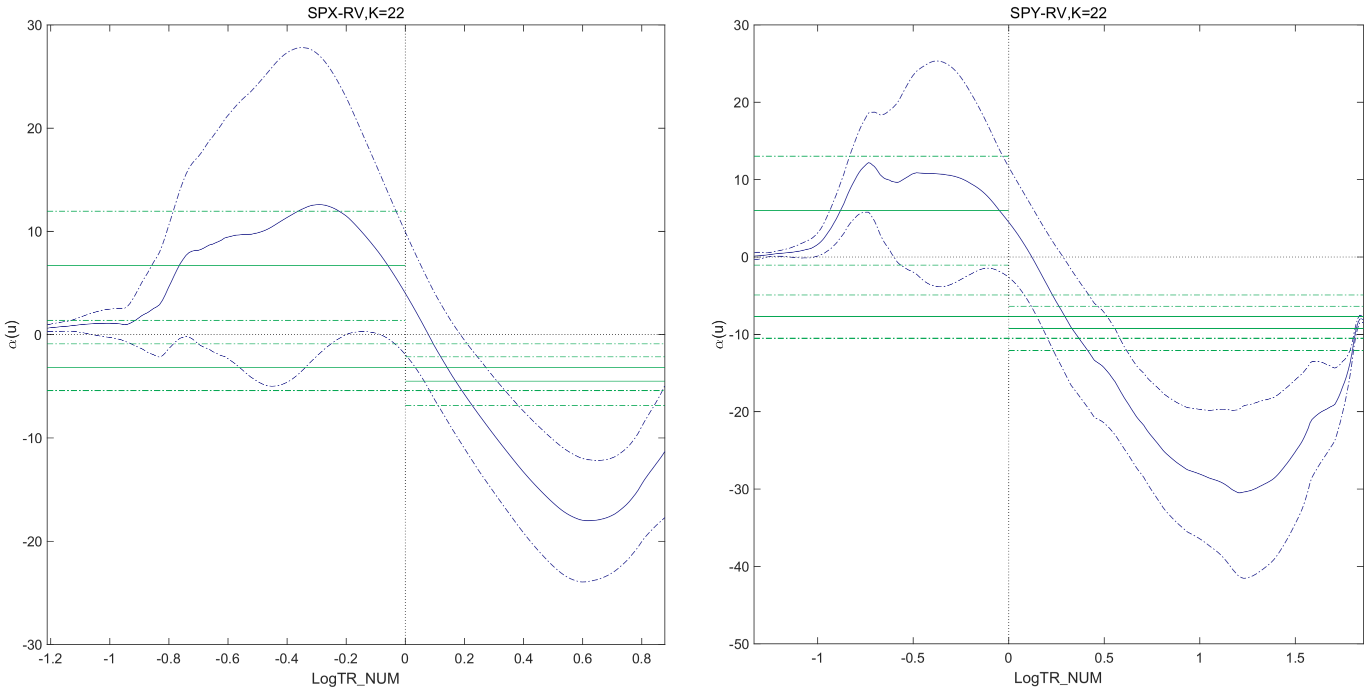

Fig. 4. The nonlinear risk premium of SPX and SPY, where the number of trades serves as the trading volume. The blue curves indicate the nonlinear relationship $\alpha(u)$ between r_t and RV_t under the impact of $LogTR\_NUM_t$ in the model (Eq. (5)). The green lines are the constant and threshold regression coefficients of RV_t in model (Eq. (8)) and model (Eq. (9)), where the half short green line on the left of zero vertical line is α_1, the other half short green line on the right of zero vertical line is α_0 and the longest green line is α_c. The blue and green dotted lines are for the corresponding 95% confidence interval lines. The data sample period is 01/02/2009–12/31/2018. The blue curves have up-down-up fluctuating patterns, indicating the nonlinear time-varying risk premium. Both constant green lines (μ) are significantly negative. Both right half threshold green lines (α_0) are significantly negative when $LogTR\_NUM_t$ is positive. Conversely, the left half threshold constant coefficients (α_1) are positive when $LogTR\_NUM_t$ is negative. However, the confidence interval of (α_1) of SPY includes zero when $LogTR\_NUM_t$ is less than zero. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

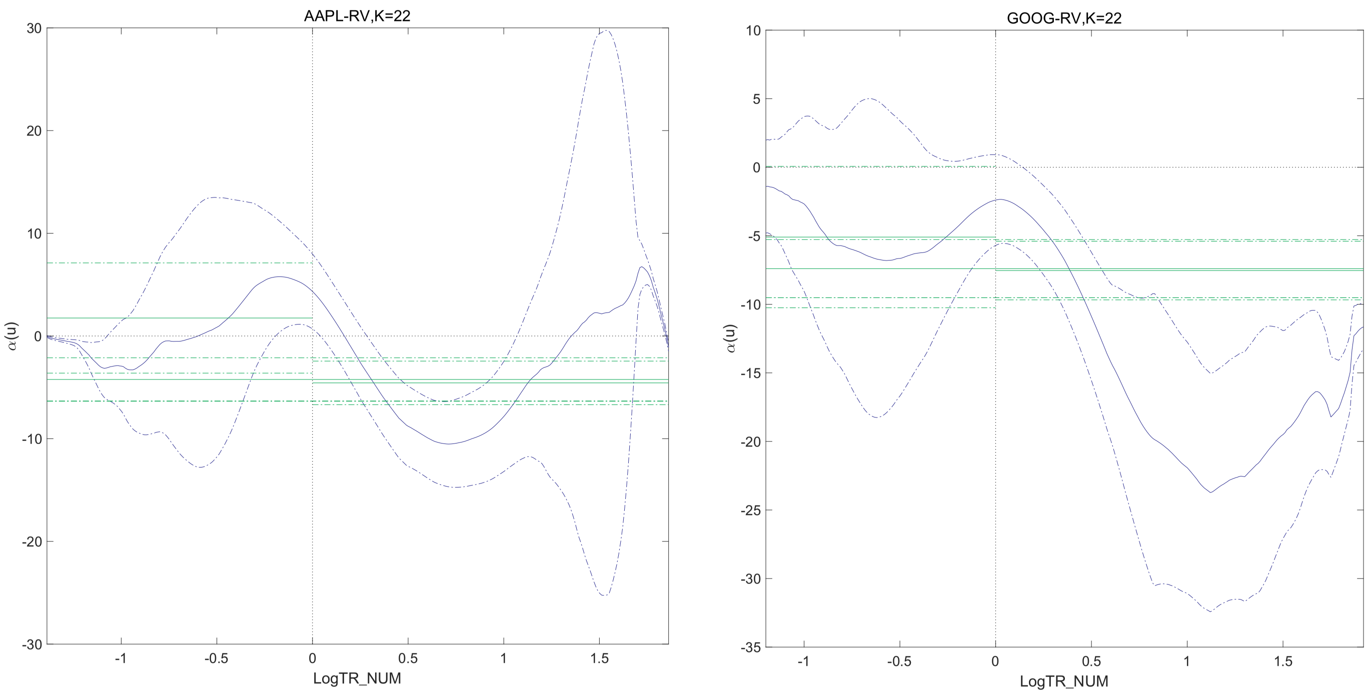

Fig. 5. The nonlinear risk premium of AAPL and GOOG, where the number of trades serves as the trading volume. The marked lines are the same as those in Fig. 4. Though not having obvious up-down-up fluctuating patterns such as SPX and SPY, the $\alpha(u)$ of AAPL and GOOG is also nonlinear. Except for the constant coefficients (α_0), the other two green lines (α_1) are insignificant from zero when $LogTR\_NUM_t$ is negative. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

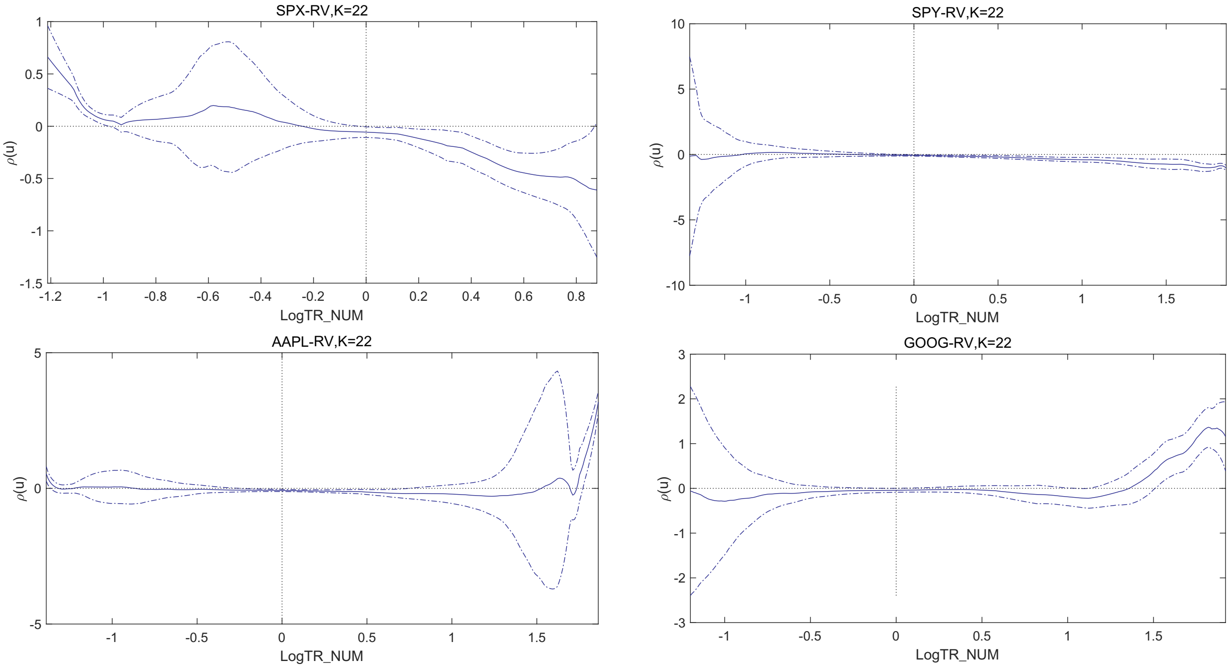

Fig. 6. The nonlinear autocorrelation coefficient ρ(·) for SPX, SPY, AAPL and GOOG, where the number of trades serves as the trading volume. Most of the time, the autocorrelations are insignificant. Those segments significant from zero are only for abnormal trading days with extreme changes in trading volume.

pattern with the increase of u_t; it is insignificantly positive on inactive trading days and significantly negative on active trading days. In fact, although there are no fixed patterns for individual stocks, it can still be concluded that the autocorrelation is most likely significantly different from zero during abnormal trading days when *LogTR_NUM* is far from zero on extremely active or inactive trading days. In other words, the autocorrelations will be zero or have small significant values on normal trading days, which is consistent with most empirical findings. The market is efficient when there is insignificant autocorrelation of returns. However, the autocorrelation can be positive or negative during abnormal trading days.

The risk-return trade-off has a fixed pattern that is first flat, then increasing and decreasing, and finally increasing as *LogTR_NUM* increases from negative to positive. This reflects the effects of investor's changing risk preference, as reflected by the trading volume or information flow. As the number of trades decreases, market participants are initially risk averse and need a positive premium. As the stocks become hotter and the premium coefficient decreases, participants behave in a risk-seeking fashion and are overconfident, yielding a negative risk premium. As trading activities continue to increase, irrationality and overconfidence begin to dominate stock sentiment, and investors tend to overreact, causing risk premiums to decrease further. However, after reaching the bottom, the extremely high number of trades may make it difficult for traders to evaluate information, which increases uncertainties and slowly increases the premium. This phenomenon is different from the logistic function of information proposed by Kinnunen (2014). Also see Appendix. Logistical functions could overestimate and underestimate the risk premium at low and high levels of information flow, respectively, which are exactly when abnormal or irrational behavior occurs. An almost linear relationship among nearby zero points also appears for both portfolio and individual stocks. If those trading days with small changes in the number of trades are treated as normal days with no new information or event uncertainty, it is reasonable to assume a linear function of trading volume for the risk premium. In other words, all the risk premiums can range from positive to negative with the levels of new information flow increasing from negative to positive. This finding is in line with the findings of Whitelaw (1994), who divided a cycle into four regions and shows that the contemporaneous correlation between returns and volatility varies from positive to negative and is not constant over time. He also found that the contemporaneous correlation can be close to zero if it is estimated over the full cycle. Furthermore, Brandt and Wang (2010) showed that the time-varying risk-return relation ranges from negative to positive, with the related variables placed in a linear parametrized equation based on Fama-French factors.

These findings explain why the results of autocorrelation ρ(·) are not equal to 0 and significant only during abnormal trading days. According to the AMH of Lo (2004), the predictability of return is "highly context dependent and dynamic", which indicates that predictability depends on changing market conditions. During normal periods, the stock market is efficient, with insignificant autocorrelation. However, it may become temporarily inefficient in abnormal periods. Cooper, Gutierrez Jr, and Hameed (2004) find similar results for short-run momentum profits. Furthermore, from the perspective of behavioral finance, predictability is related to the under-reaction or overreaction of investors; see the papers by Daniel, Hirshleifer, and Subrahmanyam (1998) and Hong and Stein (1999). If the sharp increase or decrease in trading volume is treated as an indicator of overreaction or under-reaction, the predictability of return becomes stronger as the number of trades approaches extremely low or high levels. During periods with low

Table 3
Descriptive statistics of nonlinear risk premium and autocorrelation, where *LogTR_NUM* serves as the proxy for levels of new information.

	Mean	Min	Max	Std.
$\alpha(\cdot)$				
SPX	−0.749	−17.995	12.597	10.322
SPY	−8.069	−30.492	12.203	14.982
AAPL	−1.527	−10.522	6.750	4.888
GOOG	−11.232	−23.730	−1.386	7.531
$\rho(\cdot)$				
SPX	−0.095	−0.550	0.310	0.227
SPY	−0.258	−1.030	0.155	0.310
AAPL	−0.014	−0.294	3.173	0.391
GOOG	0.048	−0.289	1.365	0.388

trading volume, investors tend to under-react to market information when facing no information or event uncertainty. Conversely, investors tend to overreact to the high levels of information during the most active periods with high trading volume. These results can help explain why some empirical findings support and contradict finance theories.

Table 3 presents the statistics of estimators $\alpha(\cdot)$ and $\rho(\cdot)$ in the model (Eq. (5)). Table 4 presents the estimated coefficients of regression models (Eq. (8)) and (Eq. (9)), where *LogTR_NUM* serves as the proxy for levels of new information. We find that all of the means of the risk premiums in Table 3 are negative, which is consistent with Table 4 and Table A.1, which give the estimation results of STAR. In contrast, almost all of the maximums of the risk premium are greater than zero, which supports previous empirical studies that find a positive relation between the conditional expected return and volatility.

These findings are not only in line with the adaptive markets hypothesis (AMH) of Lo (2004) but also have important implications for explaining the conflicts among theoretical results and empirical research. With the risk premium's nonlinear features and similar weaving patterns under the impact of trading volume, we can easily obtain the following findings. First, empirical research shows that a positive risk premium is more likely when there are more inactive trading days. This finding is also given in theoretical studies that emphasize event uncertainty or risk-averse preferences. Second, one can easily obtain a negative risk premium during active trading days. Additionally, if the theoretical studies conjecture more irrationality or behavioral bias or market inefficiency, such as information asymmetry, a negative risk premium will tend to occur. Third, most of the time, we obtain a negative risk premium if the inactive trading days and active trading days are almost equal because of the asymmetric effects of trading volume with larger absolute negative values than positive values. For this reason, we obtain a negative risk premium more often, for example, the constant negative regression coefficients in Figs. 4 and 5. Finally, we obtain some inconclusive results when our applications are built on normal trading days because these days have closing-zero risk premiums. We strongly believe that these findings offer good explanations and support both theoretical studies and empirical studies as well as their contradictions.

3.2.3. *Results for turnover as trading volume*

Similar to what was done in the previous section, we first express our findings regarding the nonlinear features of risk premium coefficients in Figs. 7 and 8, respectively, and then analyze the autocorrelation term when turnover serves as a proxy variable of information flow. Clearly, it can be seen from Figs. 7 and 8 that there is a similar weaving pattern in Figs. 4 and 5 (the blue lines).

First, the premium is positive when the turnover is lower than usual. The decline in turnover indicates a decline in liquidity compared to recent trading days. The positive coefficients imply that investors are risk averse during these periods. Investors need risk premiums to compensate for asset liquidity risks. Facing event uncertainty or no information, most investors are risk averse or neutral and make trade decisions more carefully, and the premium is close to zero or insignificant. We infer that the trade process is dominated by noise traders who are risk neutral during extremely inactive trading days.

Second, the risk premium declines from positive to negative as the turnover increases. It is believed that there may be three reasons for this phenomenon. First, as turnover increases, the risk premium for asset liquidity risk decreases. Second, the increase in turnover indicates an increase in trading activity, which may reflect an increase in the proportion of informed traders who trade many shares based on their private information or knowledge instead of the risk premium. Finally, during the active trading days, the informed and uninformed traders are both overconfident, which results in risk-seeking and chasing hot stocks. However, as the turnover increases to extreme values, the risk premium follows an upward trend. In this stage, the composition of market participants becomes much more complex, potentially entailing many speculators and individual investors or noise traders doing trading with rationality and irrationality. Information becomes more uncertain. As a result, the risk premium grows as trading activities increase at this stage. Of course, in general, there is a nonlinear decreasing trend under the impact of turnover.

Finally, the autocorrelation issue is explored. There seem to be no fixed patterns, and the sign is mixed. While we cannot make a definite inference for the trend of autocorrelation under the impact of information, we can infer that autocorrelation is most likely to differ from zero only during abnormal trading days. As shown in Fig. 9, autocorrelation is likely to be significantly different from zero at both ends of the line, which indicates predictability and market inefficiency. However, all normal trading days have

Table 4
The constant coefficients and autocorrelation, where *LogTR_NUM* serves as the proxy for levels of new information.

	SPX		SPY		AAPL		GOOG	
α_c	−3.147		−7.696		−4.233		−7.397	
	(−5.403, −0.891)		(−10.482, −4.910)		(−6.341, −2.126)		(−9.518, −5.275)	
α_0		−4.491		−9.225		−4.567		−7.535
		(−6.833, −2.148)		(−12.094, −6.355)		(−6.691, −2.443)		(−9.676, −5.395)
α_1		6.683		5.996		1.75		−5.094
		(1.400, 11.967)		(−1.042, 13.034)		(−3.614, 7.115)		(−10.252, 0.064)
ρ_c	−0.06	−0.067	−0.086	−0.101	−0.082	−0.083	−0.034	−0.036
	(−0.100, −0.020)	(−0.107, −0.027)	(−0.126, −0.046)	(−0.141, −0.061)	(−0.121, −0.042)	(−0.123, −0.044)	(−0.074, 0.005)	(−0.075,0.004)

The numbers in brackets are corresponding 95% confidence intervals.

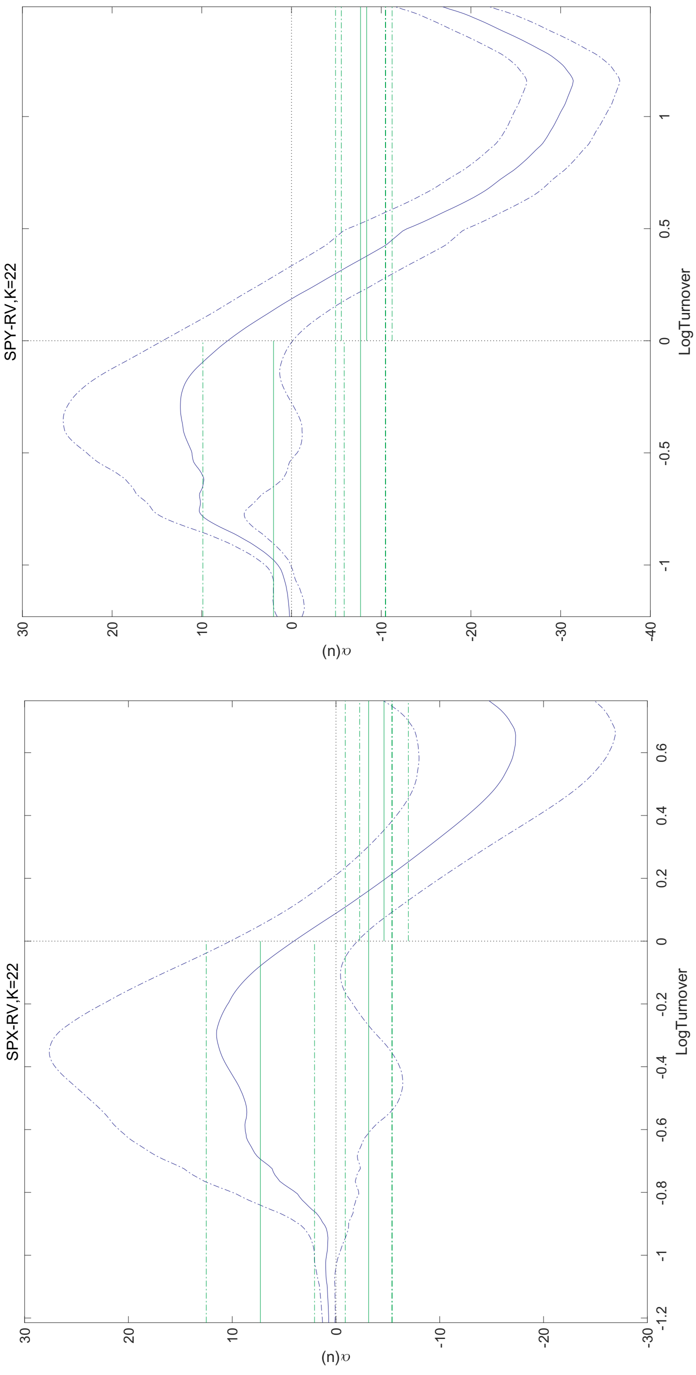

Fig. 7. The nonlinear risk premium of SPX and SPY, where turnover serves as the trading volume. The marked lines are the same as those in Fig. 4.

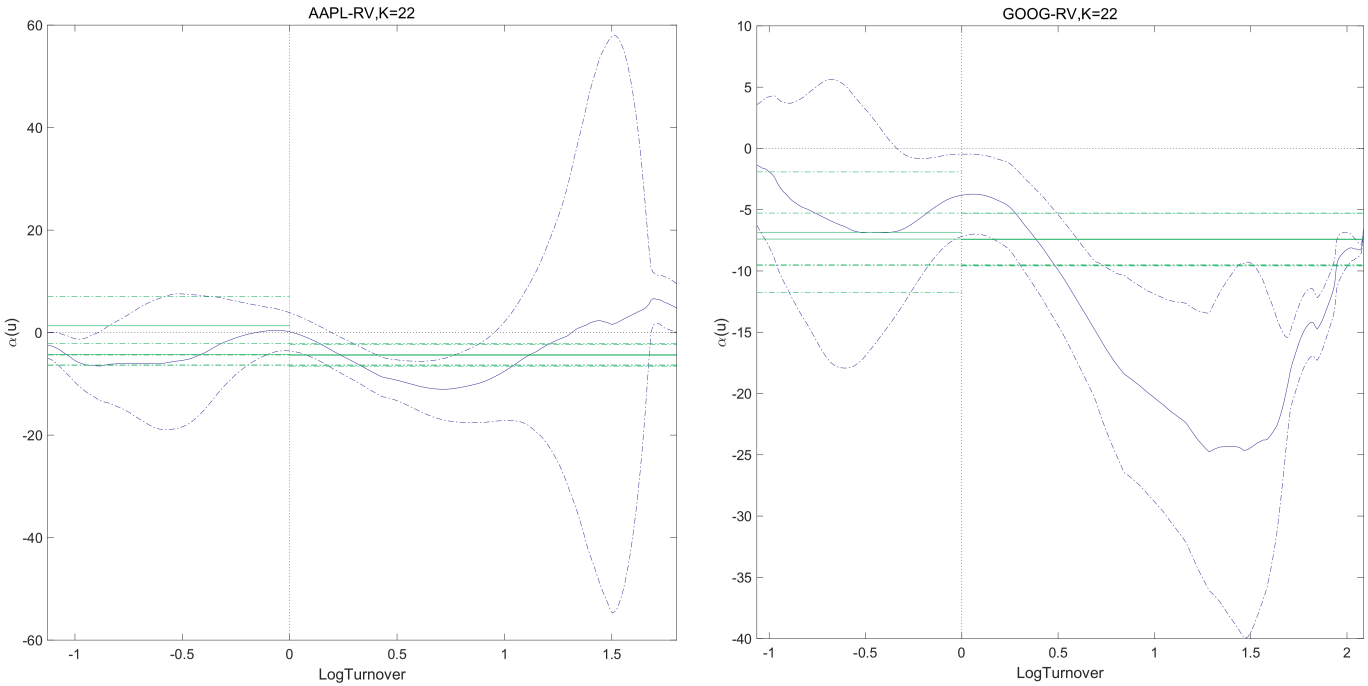

Fig. 8. The nonlinear risk premium of AAPL and GOOG, where turnover serves as the trading volume. The marked lines are the same as those in Fig. 4. The blue lines have a weaving pattern similar to that in Fig. 5. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

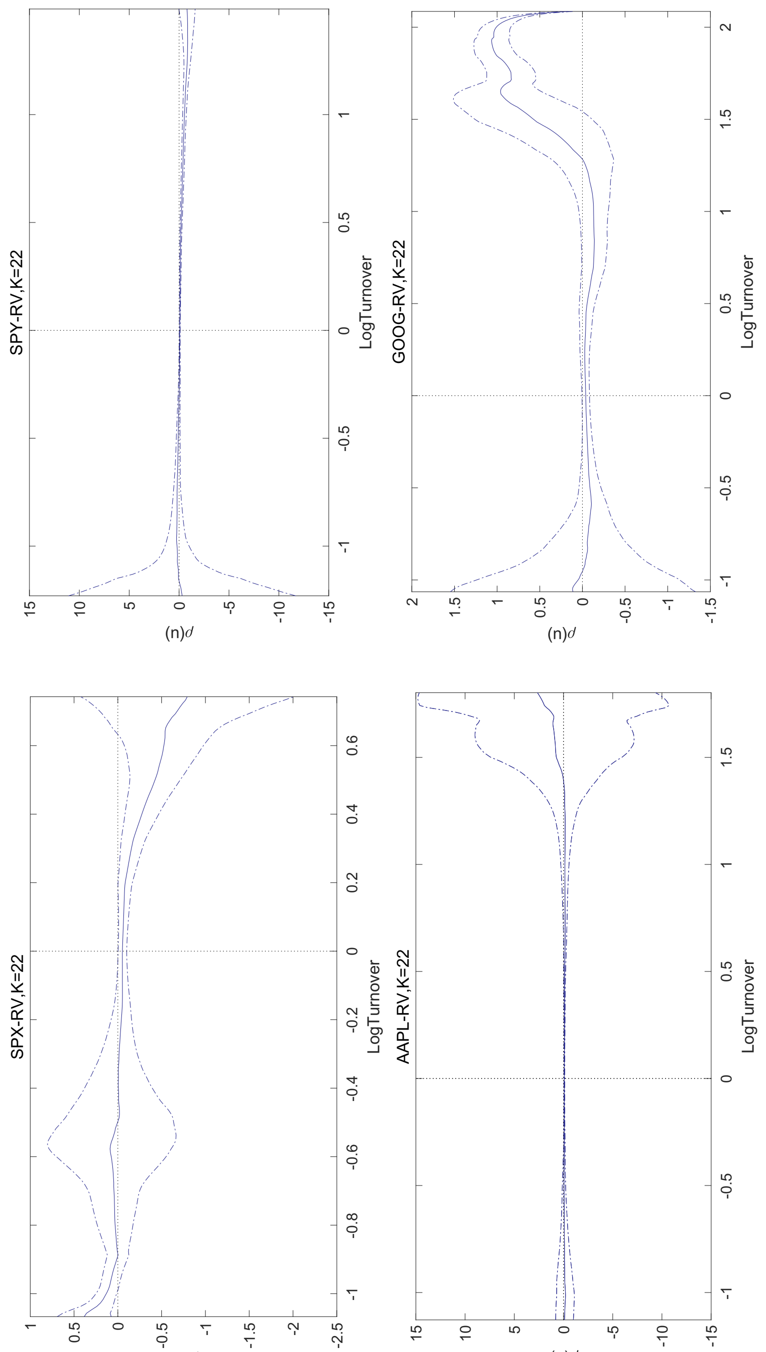

Fig. 9. The nonlinear autocorrelation coefficient ρ(·) for SPX, SPY, AAPL and GOOG, where turnover serves as the trading volume.

Table 5
Descriptive statistics of nonlinear risk premium and autocorrelation, where *LogTurnover* serves as the proxy for levels of new information.

	Mean	Min	Max	Std.
$\alpha(\cdot)$				
SPX	−0.132	−17.314	11.510	9.477
SPY	−6.166	−31.409	12.387	15.847
AAPL	−3.800	−11.051	6.634	4.549
GOOG	−11.788	−24.767	−1.347	7.546
$\rho(\cdot)$				
SPX	−0.097	−0.625	0.534	0.222
SPY	−0.195	−0.859	0.247	0.326
AAPL	0.043	−0.208	2.609	0.466
GOOG	0.133	−0.140	1.066	0.381

autocorrelation values of zero. This finding is in line with the conclusion of previous studies that there is no constant sign of autocorrelation. This further supports the findings in the previous section that inefficiency occurs on abnormal trading days, especially trading days with large turnover.

Tables 5 and 6 present the descriptive statistics of the functional coefficients and constant coefficients where turnover serves as the proxy of information. These two tables yield results similar to those shown in Tables 3 and 4, respectively.

In summary, our findings show that the contemporary relationship between return and realized volatility is nonlinear and time-varying under the impact of trading volume with an up-down-up pattern, ranging from positive to negative. Investors tend to under-react or overreact on abnormal trading days with positive or negative premiums. During abnormal trading days, the market is likely to be inefficient, as the autocorrelation is significantly different from zero. If only constant or linear models are used to analyze the dynamics of financial markets, they are prone to under- or overestimation, producing specious conclusions. These dynamic nonlinear relationships can be helpful in explaining the differences and contradictions between finance theory and the empirical findings of most studies.

4. Robustness checks

In this section, the robustness checks of our findings are conducted for the following four aspects: the sub-period samples, jumps robust with bi-power variation (BV), trading volume and trading volume de-trend. All of the robustness tests have similar results to our main findings. In fact, we also perform robustness tests with constant autocorrelation $\rho(\cdot)$ in model (Eq. (5)), using absolute returns as a measurement of volatility, a squared root of RV as well as other individual stocks except our selected ten large stocks. Because their results are similar to our main findings, we do not present them in this section, and they are available upon request.

4.1. Subperiod

In this subsection, we first examine the sensitivity and robustness checks of our findings using sub-period samples. The whole sample is split into two subsamples at the midpoint of 5 years. Fig. 10 presents the results based on the sample data from January 2, 2014, to December 31, 2018. The relationships between return and RV are similar to those above. Actually, their up-down-up weaving patterns are more obvious.

4.2. Bi-power variation

Now, the bi-power variation, introduced by Barndorff-Nielsen and Shephard (2004), is adopted as a consistent estimator for the continuous component, defined by

$$BV_t = \mu_1^{-2}\sum_{i=2}^{M}|r_{t,i}||r_{t,i-1}| \to \int_0^t \sigma_s^2 d_s, \quad \text{as } M \to \infty ,$$

where $\mu_1 = \sqrt{2/\pi}$. It is well known in the literature that BV_t is robust to jumps. Indeed, the graphs in Fig. 11 indicate no large difference from the results of Section 3.2, presenting almost the same patterns for risk premiums. However, the autocorrelation is quite different.

Table 6
The constant coefficients and autocorrelation when *LogTurnover* serves as the proxy for levels of new information. The numbers in brackets are corresponding 95% confidence intervals.

	SPX		SPY		AAPL		GOOG	
α_c	−3.147 (−5.403, −0.891)		−7.696 (−10.482, −4.910)		−4.233 (−6.341, −2.126)		−7.397 (−9.518, −5.275)	
α_0		−4.629 (−6.975, −2.283)		−8.382 (−11.214, −5.551)		−4.434 (−6.549, −2.319)		−7.44 (−9.590, −5.290)
α_1		7.285 (2.066, 12.503)		5.996 (−5.871, 9.877)		1.328 (−4.373, 7.028)		−6.839 (−11.759, −1.918)
ρ_c	−0.06 (−0.100, −0.020)	−0.068 (−0.108, −0.028)	−0.086 (−0.126, −0.046)	−0.092 (−0.132, −0.052)	−0.082 (−0.121, −0.042)	−0.082 (−0.122, −0.043)	−0.034 (−0.074, 0.005)	−0.035 (−0.074, 0.005)

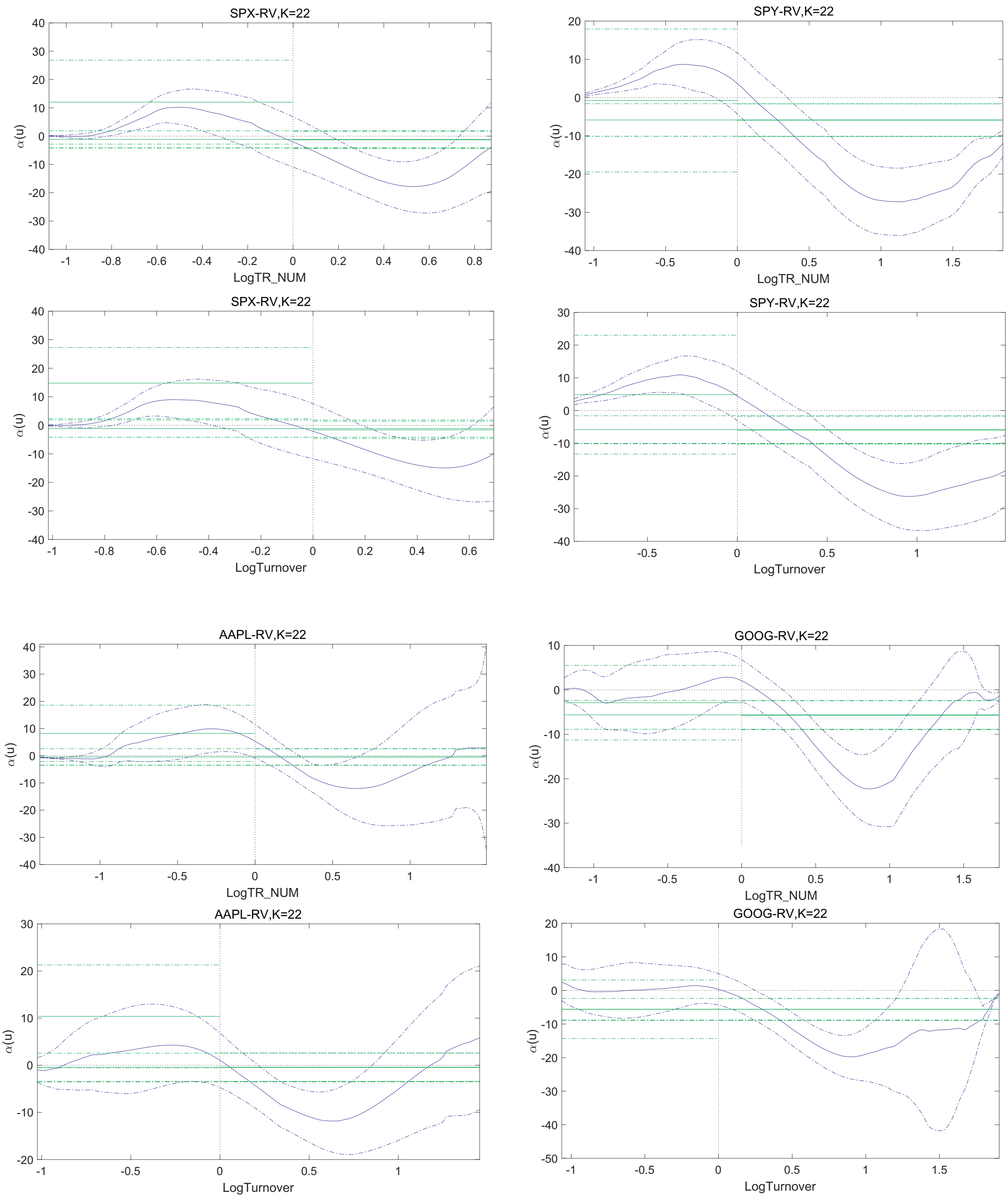

Fig. 10. Subperiod robustness checks with half of the sample period from 01/02/2014 to 12/31/2018.

4.3. Trading volume

Next, we use an alternate measure of trading volume, namely, aggregate or individual dollar volume, as in Lakonishok and Vermaelen (1986) and Lo and Wang (2000), where dollar volume is also an important measure of trading volume. Fig. 12, where dollar volume is as a measure of trading volume, presents patterns similar to our main results shown in Figs. 4 and 5.

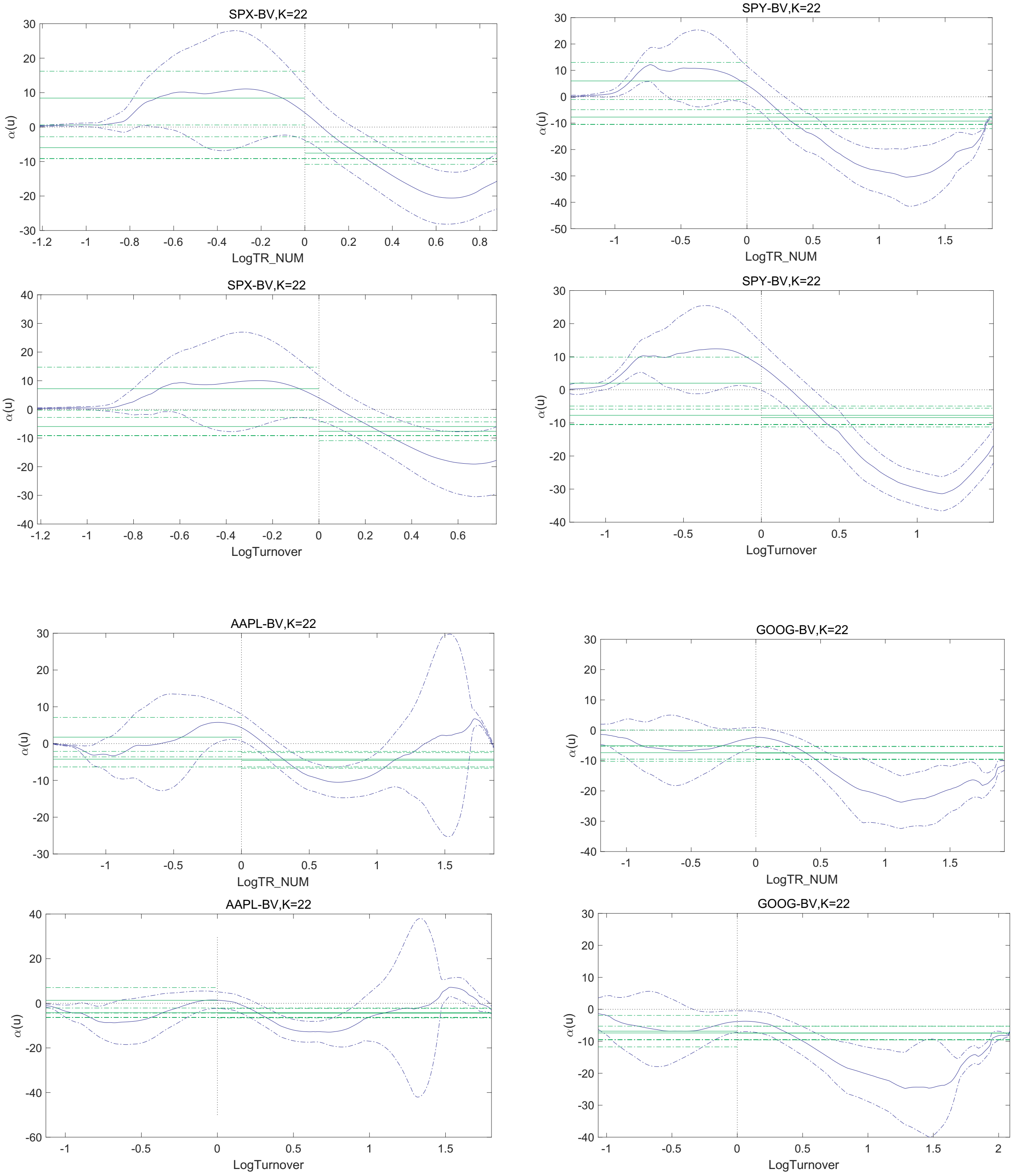

Fig. 11. Robustness checks using BV as a volatility measurement, which is robust to jumps.

4.4. Trading volume de-trend

Finally, $K = 5$ is used to de-trend the log volume by 5 days, which is the number of trading days of 1 week in (Eq. (6)) and (Eq. (7)). Fig. 13 plots the estimation results. Again, the plots clearly show that the patterns of the contemporary relationship between return and RV are robust to the de-trending method.

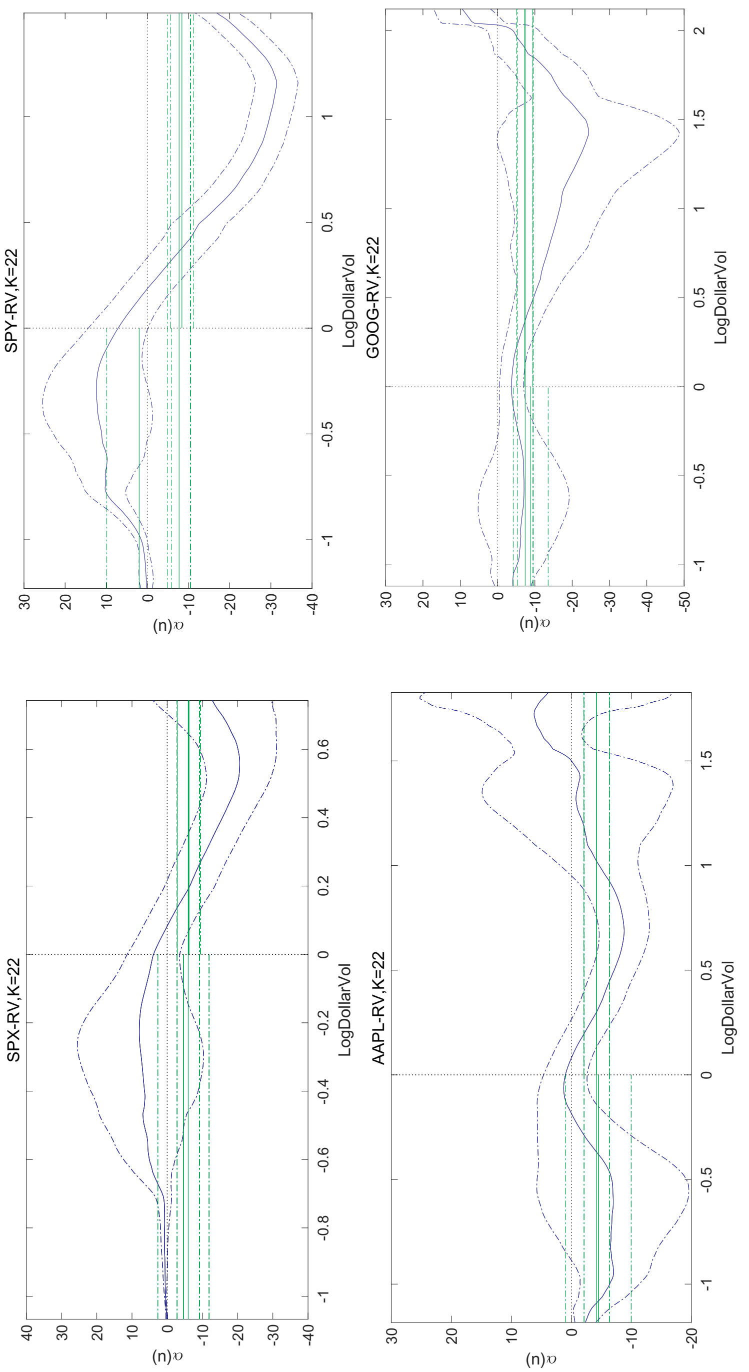

Fig. 12. Trading volume measurement robustness checks by using the dollar volume as a measure of trading volume.

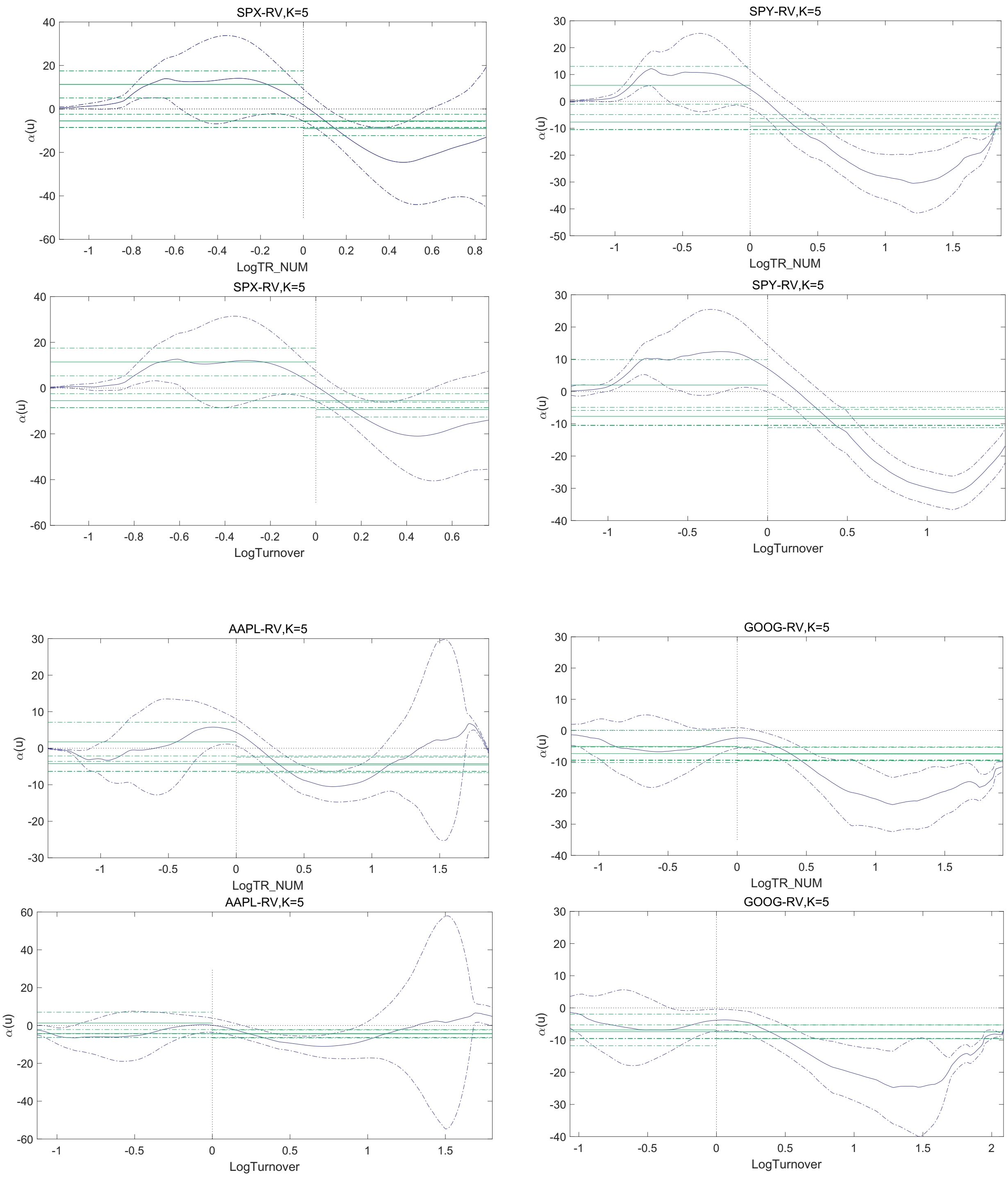

Fig. 13. De-trend robustness checks using $K = 5$ lagged days.

5. Conclusion

Despite the vital importance of the contemporary relationship between returns and volatility in finance theory and practice, there are many contradictions between the theoretical and empirical literature. In this paper, we concentrate on the nonlinear and time-varying risk premium by investigating the contemporary relationship between returns and realized volatilities under the impact of trading volume, which is an excellent proxy of information flows. We use SPX, SPY and ten large individual capital stocks and adopt high-frequency data for a 10-year period from 2009 to 2018 in TAQ. The results imply a nonlinear and time-varying relationship between returns and RV under the influence of trading volume. The risk premium can range from negative to positive. During days with low trading volume, the risk premium is positive. As the trading volume increases, the risk premium becomes negative. However, the risk premium approaches zero during the abnormal trading days when the trading activity is very low or very high. We use several financial theories to explain these observations. During days with low trading volume, stock traders are risk averse, facing no news or event uncertainty, and they need a positive risk premium for uncertainty or liquidity. As the trading volume increases on active trading days, on the one hand, the proportion of informed traders who trade in a given direction based on their knowledge other than the risk premium increases. On the other hand, traders tend to be overconfident or overreact to information flows, acting in a risk-seeking fashion. As a result, the risk premium is negative. However, the risk premium tends toward zero during extremely abnormal trading days. During extremely inactive days, this result may be attributed to noise traders, who are risk neutral and trade regardless of the circumstances. During extremely active trading days, trader proportions are much more complex, involving informed traders, uninformed traders, speculating traders, and irrational or overconfident traders. Therefore, traders tend to be risk neutral in the whole. Based on these findings, previous studies are prone to offer confusing conclusions.

Furthermore, the autocorrelation of returns is mixed and complicated, and there seems to be no fixed fluctuating pattern with regard to the impact of trading volume. In general, the autocorrelation is insignificantly different from zero during normal trading days with little change in trading volume, which means that there is no predictability of returns. This finding is in line with our commonly held views that the returns of the stock market are unpredictable. However, significant autocorrelation or inefficiency is more likely to occur during abnormal trading days. Our findings are much richer than those of studies with constant or linear assumptions of risk premiums. They are good explanations and support for theoretical studies and empirical research as well as their contradictions.

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Results of the STAR model

Similar to the model in (Eq. (3)), the following empirical model estimation is considered:

$$r_t = \mu + \varphi_t \lambda RV_t + (1 - \varphi_t)\rho r_{t-1} + \varepsilon_t, \tag{A.1}$$

where $\varphi_t = [1 + e^{\theta + \beta u_t}]^{-1}$, and $\varepsilon_t \sim N(0, \sigma^2)$. Note that φ_t is related to t through the impact of u_t. All parameters are estimated via maximum likelihood, which is implemented by the package **maxLik** in R using the BFGS optimization algorithm, and multiple different sets of initial values are used to ensure global maximum. Some main results of SPX and SPY are given in Table A.1 and Fig. A.1, respectively. One can clearly see that the varying contemporary relationship ($\lambda\varphi_t$) between returns and RV is a declining logistic curve and is almost negative and asymmetric, which is in line with the empirical findings as in Bekaert and Wu (2000) and Badshah et al. (2016). Another interesting thing to note is that φ_t is an increasing logistic curve because λ is negative and asymmetric because θ is significantly larger than zero. Thus, this means that the impact of the first-order autoregressive term is a decreasing function of u_t (trading volume). Namely, as the trading volume increases, market efficiency increases and investors behave more rationally; see the illustration in (Eq. (3)) in Section 2.1.

Table A.1
Estimation results for SPX (the top part) and SPY (the bottom part) from STAR model.

	Estimate	Std. error	t-value	*p*-value
SPX				
LogTR_NUM				
μ	0.0004	0.0002	2.2220	0.0263
λ	−18.6100	1.4890	−12.4970	0.0000
ρ	−0.0543	0.0215	−2.5200	0.0117
β	−6.6250	0.7420	−8.9280	0.0000
θ	3.7630	0.0246	153.2860	0.0000
σ	0.0084	0.0001	70.6010	0.0000
LogTurnover				
μ	0.0003	0.0002	2.0530	0.0401
λ	−15.0800	0.5246	−28.7510	0.0000
ρ	−0.0509	0.0213	−2.3950	0.0166
β	−14.1900	0.9089	−15.6150	0.0000
θ	5.8190	0.5279	11.0230	0.0000
σ	0.0084	0.0001	70.5900	0.0000

(*continued on next page*)

Table A.1 (*continued*)

	Estimate	Std. error	t-value	*p*-value
SPY				
LogTR_NUM				
μ	0.0006	0.0002	3.6790	0.0002
λ	−10.9600	0.7502	−14.6080	0.0000
ρ	−0.0557	0.0301	−1.8500	0.0643
β	−11.9900	0.2290	−52.3560	0.0000
θ	1.4410	0.0348	41.3860	0.0000
σ	0.0079	0.0001	70.6230	0.0000
LogTurnover				
μ	0.0005	0.0002	3.0580	0.0022
λ	−10.0100	0.7468	−13.4040	0.0000
ρ	−0.0601	0.0257	−2.3440	0.0191
β	−32.0800	0.1259	−254.7520	0.0000
θ	7.9610	0.7475	10.6510	0.0000
σ	0.0079	0.0001	70.5970	0.0000

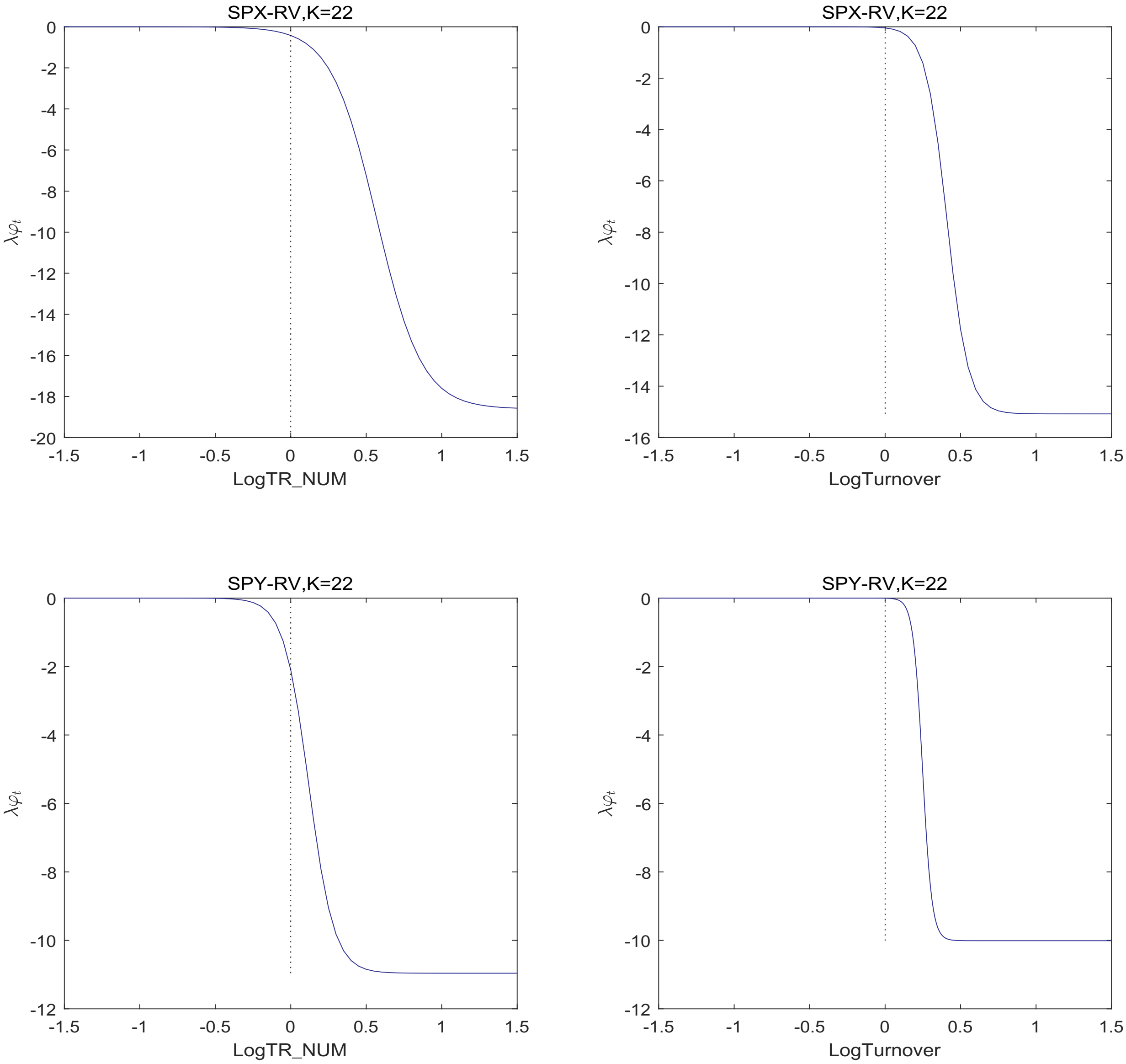

Fig. A.1. Plots of $\lambda\varphi_t$ in model (A.1) for SPX (the top panel) and SPY (the bottom panel), respectively, with y-axis as $\lambda\varphi_t$ and x-axis as *LogTR_NUM* (the top panel) or *LogTurnover* (the bottom panel).

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